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# TRANSACTIONS

OF THE

# AMERICAN INSTITUTE

OF

# ELECTRICAL ENGINEERS.

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# AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York, January 26th, 1898.

The 121st meeting of the INSTITUTE was held this date, at 12 West 31st Street, and was called to order by President Crocker at 8:30 P. M.

THE PRESIDENT:—The Secretary has some announcements to make.

THE SECRETARY:—At the meeting of Council this afternoon, it was decided to hold the General Meeting of the INSTITUTE next summer at Omaha, if satisfactory arrangements could be made. This is in accordance with the resolution passed at the General Meeting at Greenacre, favoring such action. The fixing of the exact date and the perfecting of the arrangements were left to the Executive Committee.

The following associate members were elected:

| Name. | Address. | Endorsed by |
|---|---|---|
| BANGS, CHAS. R. | Special Agent, American Telephone and Telegraph Co., 15 Dey St., New York. | F. A. Pickernell. A. N. Mansfield. J. J. Carty. |
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| CROCKER, EBEN CLINCH | Electrical Engineer, American Ordnance Co., 29 Harriet St., Bridgeport, Conn. | Jos. Wetzler. R. T. Lozier. Chas. Blizard. |
| ELIAS, ALBERT B. | Electrician, Davis Coal and Coke Co., Thomas, West Va. | Chas. H. Davis. A. E. Braddell. M. J. Wightman. |

| | | |
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| HELLICK, CHAUNCEY GRAHAM | Electrical Engineer, The Chicago Telephone Co., residence, 193 Dearborn Ave., Chicago, Ill. | J. J. O'Connell.<br>A. V. Abbott.<br>F. J. Dommerque. |
| MORGAN, JACQUE L. | Electrical Inspector, Kansas City Fire Dep't.; residence, 1702 Locust St., Kansas City, Mo. | Edwin R. Weeks.<br>Sam'l. Insull.<br>Axel Ekstrom. |
| PROSSER, HERMAN A. | Electrician, Baltimore Copper Smelting and Refining Co., Keyser Bldg.; residence, 1222 Madison Ave., Baltimore, Md. | F. B. Crocker.<br>G. F. Sever.<br>W. H. Freedman. |
| SWANN, JOHN JOSEPH | Assistant Editor, *Engineering News*, 220 Broadway; residence, 847 West 34th St., New York. | H. J. Ryan.<br>Edw. L. Nichols.<br>Ernest Merritt. |
| SWOOPE, C. WALTON | Instructor, Electrical Engineering, Spring Garden Institute; residence, 12 North 38th St., Philadelphia, Pa. | Wm. S. Aldrich.<br>A. J. Rowland.<br>Wm. C. L. Eglin. |
| TACHIHARA, JIN | General Electric Co.; residence, 106 Union St., Schenectady, N. Y. | C. P. Steinmetz.<br>Ernst J. Berg.<br>Eskil Berg. |

I presume that many of the members have read in the daily papers the sad intelligence of the death of one of our distinguished fellow members, Mr. O. B. Shallenberger, for many years connected with the Westinghouse Electric and Manufacturing Company, as consulting engineer.

Mr. Shallenberger died at Colorado Springs, on January 23rd. His funeral will take place on Saturday next. Mr. Shallenberger was thirty-eight years of age and was a graduate of the United States Naval Academy. He became identified with the INSTITUTE in 1888, and was transferred to full membership the same year. I presume that there are many of the members present who have had the pleasure of his personal acquaintance, and as soon as particulars of his death are received a sketch of his life will be published in the TRANSACTIONS.

*A Topical Discussion at the 121st Meeting of the American Institute of Electrical Engineers, New York, January 26th, 1898. President Crocker in the Chair. Chicago, Manager B. J. Arnold in the Chair.*

# THE STANDARDIZING OF GENERATORS, MOTORS AND TRANSFORMERS.

## (A Topical Discussion.)

THE PRESIDENT:—Gentlemen, in taking up the discussion, which is the object of this meeting, it might be well to make a few statements as to what it is intended to cover, and the scope of it, as that matter has not been understood by very many of the members. The general desirability of standardizing electrical apparatus has often been spoken of and seems to be very generally believed in. The feasibility of it, or the policy of it, is another matter. If such standardizing is to be effected, this body, THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, is undoubtedly the only one which is competent to decide such a question. As to precedents, I think we have many excellent ones. I have here volume vi. of the *Transactions* of the *American Society of Mechanical Engineers*, for the year 1884, and in it is a very elaborate report of the Committee on Standard Methods of Steam Boiler Trials. This, I believe, was the first action of the *American Society of Mechanical Engineers* on this general line of work, and since then a number of other standards or standard methods of practice have been recommended by it. Other bodies, such as the *American Society of Civil Engineers*, and our sister engineering societies in England, have frequently taken such action. It is very common on both sides of the Atlantic. Therefore we should be doing nothing radical or unusual in establishing such standards provided they are found desirable. It might be said, of course, that since any hard and fast adoption or acceptance of a standard would not seem to be desirable, all these bodies, particularly the *American Society of Mechanical Engineers*, have very carefully confined their action to a recommendation of certain standards, and they have taken particular pains that it shall not appear that the absolute adoption of a certain standard is imposed upon anyone. The individual members of the INSTITUTE or any other society, are not bound, morally or legally by any such recommendation,

although they would naturally follow it. It is simply left to any person to follow it, as he sees fit, whether he is a member or not, and it is merely for the convenience of the members and the public generally that any such standards would be recommended. I think those points may have been misunderstood very naturally, and might act as a very serious obstacle in carrying out any such work. Now, as to what the standards should be and how far any such action should go, that remains, of course, to be seen, and I think the desirability of such standards, what they shall be, and how far they shall go, would be the natural subject of the discussion this evening. The general question of policy and advisability is also a proper point for the members to consider.

I would call upon Mr. E. W. Rice, Jr., to open the discussion.

MR. RICE:—Before proceeding with the few remarks that I have prepared, I wish to say that I undertook to open this discussion with considerable reluctance, seeing that I would not have time, perhaps, to do the subject justice. However, I hope that you will make proper allowance for my remarks, knowing that I have had very little time in which to prepare for this subject. I have also not had an opportunity to discuss the matter with the Executive Committee, and so do not have clearly in mind the proposed limits of the discussion.

The subject which has been selected for discussion this evening, "Standardizing of Generators, Motors and Transformers," covers in its various aspects such a large field that it would be difficult to attempt anything more than a very cursory review.

It may include the standardizing of the various types and sizes of apparatus, or the methods of rating, or of testing, or it may define such terms as regulation, efficiency, or set the limits of heating and of sparking; or again, in alternating current work, standardize the frequency.

I assume that the chief object of a meeting of this kind is to discuss the subject thoroughly, and that if as a result of such discussion it is considered that any useful purpose can be attained, the matter will be referred to a carefully selected committee with instructions to consult with the various interested parties and report its conclusions with recommendations to the INSTITUTE at some future date.

The advantages of establishing certain standard types and sizes of apparatus are apparent and have been fully recognized, particularly in this country, from the earliest days of electric lighting, which marked the beginning of the present electrical industry. Such standard lines have been arbitrarily determined by each of the different manufacturing companies, and have been built to conform to certain specifications, and tested in accordance with methods developed by experience. Competition has tended to increase the number of standard sizes and the development of the industry has added enormously to the variety of apparatus in use. I think it is generally recognized that the adop-

tion of such standards by the manufacturers and their acceptance by the public has been one of the greatest factors in the phenomenal development of our industry in the United States, and is chiefly responsible for the present enviable position of our manufacturers of electrical apparatus as compared with other countries. It has enabled our manufacturers to produce the standard lines selected, at phenomenally low costs, and has also resulted in an equally great improvement in the quality. Large sums of money have been expended in special tools, which, while reducing the amount of labor per machine to a remarkably low figure, has, by the very means adopted for such economical production, improved the quality, because the manufacture of large quantities of duplicate apparatus has stimulated the manufacturer to spend these large amounts for tools and to organize, elaborate and perfect methods of manufacture. The history of the electrical business in this respect, of course, only parallels that of the bicycle, the typewriter, the sewing machine and other similar industries.

The importance of maintaining and extending, if possible, the policy which has enabled this country to sell apparatus in competition with other countries, frequently in the face of protective duties, is probably so thoroughly appreciated as to require no special pleading to obtain for it a hearty endorsement by those interested in the advancement of our industry.

Results such as have been mentioned are largely dependent upon the manufacture in great quantities of exactly duplicate pieces of apparatus, a condition which can only be practically realized where the apparatus is manufactured under the control of one corporation. It therefore seems evident that even if it were possible for all the interested parties to agree upon certain standards, the advantages above mentioned would not be fully realized. The chief beneficial effect of such an agreement would be to limit the demands for special apparatus which may have to be manufactured by comparatively expensive and less perfect methods, and to increase the demand for the standard apparatus.

I merely mention the above well known condition to illustrate in a general way the value of standards where they can be adopted, and to show that the manufacturer, while undoubtedly animated by selfish motives in advocating standard apparatus where possible, is at the same time advancing the best interests of the industry as a whole.

However, I think that for many reasons it would be found impracticable and probably not for the best interests of the business to establish any such standard sizes or lines of apparatus which would be equally satisfactory to the designing engineer, the consulting engineer, the manufacturer and the user.

Moreover, I do not suppose this INSTITUTE would care to attempt to introduce any standards which did not admit of definite determination or which the rapid evolution of the business would soon render useless.

There are, however, certain features of the subject under discussion which it seems to me could and should properly be considered by this INSTITUTE. These features must be considered in every commercial transaction.

I refer to the terms used to define certain characteristics of motors, generators and transformers, and the method of determining such terms. This subject has been considerably discussed of late, and was ably set forth in a recent paper by Mr. S. D. Greene, read before the New York Electrical Society; which society has in consequence invited this INSTITUTE to consider some plan for standardizing certain apparatus. I think that all of the features mentioned by Mr. Greene as requiring standardizing in practice are important, and while I am not entirely sure that all admit of such standardizing, I believe they should all be considered by this INSTITUTE. I shall therefore follow very closely Mr. Greene's list of subjects, adding one or two that have occurred to me upon a brief consideration. I shall also endeavor to give additional facts bearing upon these subjects.

1. Definition of efficiency.
2. Heating limits and methods of determining same.
3. Regulation, as applied to D. C. generators, to motors, to transformers, to A. C. generators, A. C. motors, etc.
4. Sparking in D. C. machinery.
5. Insulation—methods of testing same.
6. Rating.
7. Frequency of alternating current machinery.

1st.—*Efficiency.* The efficiency which is of practical value is the so-called commercial efficiency, that is, the ratio of the energy input to the useful energy output, thus accounting for all losses. In early days the term "electrical efficiency" was frequently confused with the term "commercial efficiency," and even at the present time the confusion sometimes occurs. Even the term "commercial efficiency" needs more exact definition as it is customary in direct coupled machinery, that is, machinery coupled directly to a prime mover, such as a steam engine, water wheel, gas engine, etc., to charge the mechanical losses, friction and windage, to the prime mover. In belted machinery the mechanical friction losses are charged against the electrical machinery. There are excellent reasons for the existing practice, but such practice at least needs standardizing.

2d.—*Heating.* While it would be obviously impracticable, even if it were desirable, to adopt any definite number of degrees increase of temperature above the air as a standard, because of the different conditions of service and design of apparatus, it would nevertheless be desirable to define the methods of determining any increase of temperature. Increase of temperature is now sometimes measured by thermometer and sometimes by in-

crease of resistance of the copper conductors. It is evident that in some cases the first method may be better, in other cases the latter. I think that the standardizing of the proper methods in all cases would be desirable.

3d.—*Regulation.* I omit in this connection any reference to direct current machines, and select for illustration alternating current apparatus. In alternating current generators the term regulation has now several different meanings. First, the percentage of drop in pressure at the armature terminals from no load to full load with no load excitation of field. Second, the percentage increase of voltage from full load to no load with full load excitation. Third, the percentage increase in field ampere-turns required to maintain constant potential at armature terminals from no load to full load. Fourth, percentage decrease in field ampere-turns from full load to no load to maintain constant potential at armature terminal. Fifth, the relation between normal full load current delivered by the machine and the current produced on short circuit with an excitation of the field corresponding to full normal load. All of the above definitions and several others, I believe, are in use among engineers.

4th.—*Sparking*, or rather the limits between which freedom from sparking shall be demanded in commutating machines, is of importance to all parties. It is admittedly an extremely difficult matter to standardize. I doubt if any entirely satisfactory scientific definition of the terms used commonly in discussing such matters can be found. The subject is, however, worthy of consideration.

5th.—*Insulation breakdown tests.* It is becoming common to specify that the insulation of a generator or motor shall withstand certain breakdown tests by being subjected to a pressure considerably higher than that of operation. While it is possibly too early, in view of the rapid development of the art to determine definitely the limits and exact conditions of such tests, the question could very profitably be discussed and perhaps some recommendation of value made.

6th.—*Rating of alternating generators.* It is customary in this country to rate alternating current generators on the basis of volt-amperes output. Abroad in certain instances the rating is based upon the volt-amperes output multiplied by an assumed cosine of angle of lag, say .8. It may be desirable that the method adopted in this country should be standardized.

7th.—*Frequency of alternating current machinery.* Alternating current machinery is being built for frequencies of 25, 30, 33, 40, 50, 60, 66, 125 and 133 in this country. While excellent engineering reasons have presumably in every instance dictated the selection of each of the above frequencies, it is probable that the number selected for standardization need not be over three or four at most. In such event the engineer would still be able to select from the standards adopted, a frequency which would

properly meet the various conditions of generation, transmission, and service, etc., which he is called upon to solve. The net result would undoubtedly be beneficial to all parties—the engineer, the manufacturer and the consumer.

In the above remarks I have not attempted to do more than give an extremely brief review of certain phases of the subject under discussion. The subject is too large to be treated exhaustively at this evening's meeting even if one had the time and ability. In concluding, I wish to repeat the suggestion advanced in the beginning of my remarks, namely, that no practical advance is liable to be made in the consideration of these important questions unless the matter is referred to a small committee of members of this INSTITUTE. This committee should be instructed to confer with manufacturers, consulting engineers and prominent users of electrical apparatus, and report its recommendations to this INSTITUTE.

MR. W. A. MOSSCROP, (Communicated):—Mr. Chairman and gentlemen: there could hardly have been selected for discussion by the INSTITUTE's members a subject more broad in its scope.

To standardize generators, motors and transformers will require the formulation of specifications covering:

1st. The physical properties of the materials used in the construction of their several parts.

2nd. The proportions of general features that are common to the product of the different shops in each class.

3rd. The design of special details of construction for the different sizes of the different classes.

4th. Methods for testing.

The statement is axiomatic, that the formulation of specifications covering these points is the duty of the engineers.

The question that presents itself is, whose engineers—the manufacturer's or the buyer's?

The decision of this question should be governed by a determination as to who is to be held responsible for possible failures in the performance. If it is to be the engineer for the buyer, then it is his right to formulate specifications covering such features as in his opinion will ensure successful performance. If, on the contrary, it is to be the manufacturer, then he should be allowed a voice in the formulation of specifications, covering the details of design and manufacture.

When there is under consideration the construction of work of special magnitude or of special requirement, the buyer's engineer will in general make the specifications. He will best serve the interests of his employer if he gives due consideration to all the manufacturing experience he can get. When he finds conflicting statements, his is the duty of deciding so that his employer will get the best and the most for his money.

When we consider the design of details of construction of machinery for the general market, which calls for standard sizes,

they are found to be matters whose choice is largely governed by shop practice, convenience and cost. Many of them are born of shop experience based upon theory. As such they are matters belonging to the manufacturer, and it will be certainly impossible for the INSTITUTE as a body to undertake the formulation of specifications covering them.

There are, perhaps, some general features of quality, design and test, for which standard specifications can be formulated, but it seems hardly possible that all the conditions governing the design of electrical machinery have yet reached the stage where one standard can be adopted for all manufacturers. It would seem that the certain way to obtain standards covering these points would be for those who are in possession of facts of interest and value in this connection to present them before the INSTITUTE by papers or by communication.

The method by communication possesses features of considerable merit, and its extension would result in bringing into closer touch with the members in New York and Chicago those who reside at a distance from these two cities. In this way standards would be obtained by virtue of a system of natural selection in which the fittest only would survive.

Some manufacturers, of course, object, and very properly too, to their engineering force presenting before the engineering societies new points within their experience, because their general publication might lower their value to a private holder.

We are, however, permitted to enjoy the fellowship of some of the brightest minds in the engineering profession who are connected with manufacturing interests. Much of their knowledge will be available to us as they find that its publication will not conflict with the interests to which they owe their first duty. And engineers who receive compensation for their services owe their first duty, in this respect, to those who employ them.

It is the manufacturer's duty to standardize his product. It is our function to help him and his employees to the attainment of the knowledge necessary for the successful manufacture of electrical machinery, and to help the buyer to make a proper selection. It is our business to bring out the facts by means of papers, communications and discussions, and it is the business of the manufacturer and the buyer to apply them. Those facts that commend themselves generally will be generally adopted, and will become standard by virtue of their fitness, and no other can be forced. If, however, there is a general belief that by formulating specifications covering some of the general requirements of the machinery under discussion we would advance the interests of the science with which we are connected, there can be most certainly no objection to our doing so. The subject needs, however, to be restricted before a plan can be outlined.

MR. ROBERT T. LOZIER:—I think the subject is an extremely broad one, but one that would be better handled in a small com-

mittee that could consider it in all its different phases. I think the general limitations are pretty well established for commercial purposes. I am speaking now only of direct current apparatus, not of alternating. I think we are doing very well at present on the existing standards. There are some details that would undoubtedly develop when the matter is placed in the hands of a committee, but any movement in that direction undoubtedly would result in a great deal of good to both manufacturers and consulting engineers—and, incidentally, of course, to purchasers.

THE PRESIDENT:—The appointment of a committee would be the natural action to be taken as the result of this meeting, but I think the committee would be very glad to have all the light possible upon the subject, and particularly the sense of the meeting, the general sentiment in favor of or against this idea, and how far it can go. I think the committee can take care of the details of such a question, but it is anxious to know the general sentiment which is very hard to get, except from a meeting. So the more light that is thrown on this subject, the more opinions expressed, the easier and more successful will be the work of the committee.

I will call upon Dr. Kennelly to speak upon this subject.

DR. A. E. KENNELLY: Mr. Chairman:—I think that the remarks we have listened to from Mr. Rice are so excellent that they require little amendment, at least on my part. I think the work of the INSTITUTE in regard to standardization should lie in the line of definitions, and not in the line of saying what should be, or should not be good apparatus, or what should or should not be good standards of apparatus. I think there are a good many phrases which are used which have different meanings, and which lead to a great many disputes, and if those phrases and definitions were formulated and standardized by the INSTITUTE, they would be of great assistance to the business and to all concerned. I think that is the proper province of an institution such as ours, in assisting and developing the business which it is our pleasure and advantage to promote. The questions of what regulation should mean, what temperature elevation should mean and how it should be measured, what efficiency should mean, etc., are eminently practical and proper technical questions which I think might save a great deal of trouble and dispute if they were settled and defined with the recommendation of the INSTITUTE. They would facilitate progress and enable people to understand each other with greater precision than they do at present. But I should be inclined to regard with disfavor any attempt made to standardize apparatus in any other way then by the process of evolution among business interests.

THE PRESIDENT:—This report that I referred to I think is of sufficient interest and application to the present case to justify me in speaking of it further. The committee that made the report consisted of Messrs. William Kent, J. C. Hoadley, R. H.

Thurston, Charles E. Emery and Charles T. Porter. The report itself is eleven pages long, and is followed by a code of rules for boiler tests, of forty pages, going into great detail and specifying just how these tests should be made; and while we have no such complicated problem as that, fortunately for us, it shows that it is common practice in that important profession and one that has produced successful results, to standardize the methods of test even to the extent of going into great detail, and our simple problem of deciding what method should be used in measuring temperature rise is a very small matter compared with the elaborate details given in this code of rules here, and it would therefore be safe for us to go into details. A general statement, and fairly specific rules as to how these tests should me made is desirable, particularly when it is remembered, as I have pointed out, that they are mere recommendations; as to whether anything more than recommendations as to methods is to be given is a point I think that ought to be considered, because I think that would be a very important one in the deliberations of the committee. If the general sense of the INSTITUTE is unfavorable to it, the committee would not have to consider that question at all; but if that question is brought in, it will be very difficult for them to decide just how far it is advisable to go. Mr. Rice suggested that three or four different frequencies would cover most ranges that were required in practice; whereas, as a matter of fact, we have many more than that. It would be interesting to know whether the various speakers would consider that a proper matter for standardizing. I think we all are somewhat hampered with the notion that this is going to be an absolute requirement from which there is no escape. There is nothing in these recommendations—if they are ever made—to prevent anyone from making another frequency. It will simply be an influence against multiplying frequencies, which in my opinion are absolutely unnecessary. Personally, I confess, I would like to see the standards adopted—not forced on the public, but simply adopted for mutual convenience, in the same way that standard screw threads are adopted—not that you cannot make another screw thread if you see fit, for certain purposes; but it is for the common convenience of all concerned that certain standards should be adopted.

MR. TOWNSEND WOLCOTT:—If I understand the situation, there are two ideas about standards—one, recognized standards in methods of testing; and the other, standard apparatus itself. Is that correct?

THE PRESIDENT:—That seems to be the way the discussion shapes itself.

MR. WOLCOTT:—I think recommending standard methods of testing is a very good idea. So far as the apparatus is concerned, I think it will eventually standardize itself, in the same sense in which you have been speaking; that is to say, we have standard

voltage for constant potential continuous current. We have a so-called 110, which may be anything up to 125. If anybody wants another voltage, he gets it, but there are a great many more machines built with that, and the double voltage, 220, than there are of the intermediate ones. Dr. Kennelly remarked about the commercial conditions of the case. I think that is what determines it, and I think these odd voltages are disappearing. I know they are discouraged by manufacturers. If asked for a machine of an odd voltage for any purpose now, the manufacturer will always tell you: We can give you that machine if you want it, but it will cost more; and that is about as strong an argument with most people, to stick to standard voltages, as I can think of—they have got to pay more for the others.

THE PRESIDENT:—If there is that influence that Mr. Wolcott speaks of, that anything but standards, after they are adopted, would naturally cost more, and therefore their use discouraged, that should be the fact, but not always is the fact. I think there is a tendency on the part of a good many people to prefer something that is not standard for some reason that is entirely unintelligible to me, and it is desirable, I think, to protect ourselves against such individuals that such standards should be adopted. The general question, of course, as Mr. Wolcott says, seems to divide itself into methods of testing and the standardizing of apparatus. There seems to be a nearly unanimous approval of standard methods of testing, but considerable doubt, in fact unfavorable opinion, in regard to standardizing apparatus.

DR. CARY T. HUTCHINSON:—I think, with the other speakers, that there is no question as to the advisability of doing something in this direction. The difference of opinion seems to be how far we should go. I doubt the advisability of attempting to establish standard methods for testing different machines. Such things, I think, take care of themselves. On the other hand, I do think that standards of performance should be established, and that incident to this, the meaning of such terms as "regulation," "efficiency," "heating limit" and others should be clearly defined. As the matter now stands, many electrical manufacturers will accept any specification from engineers, hoping, in case they fail to comply with its terms, to clear themselves by some dispute on the definition of the terms employed in the specification. Hence I think we would better specify conditions of performance, and define clearly the terms involved, rather than attempt to outline standard methods of testing.

There is by no means an agreement of practice between the different electrical manufacturers, even in such old matters as the design of small direct current machines. I have recently found the greatest diversity in machines of this type made by the different manufacturers.

I agree with Mr. Rice that this work should be done by a committee. I think that this committee should be a small one,

composed of members who can get together conveniently and frequently. In other words, that it should be a working committee rather than an honorary one. Moreover, I do not think that any electrical manufacturing companies should be represented on this committee. To have all of them represented would make the committee cumbersome; to have one or two represented, and exclude the others, would prejudice the work at the outset. I think the committee should cultivate the most friendly relations possible with the different manufacturers from whom, of course, it would be necessary that the committee should get much data and information. Representatives of the different companies should be invited to consult with the committee, and it is highly probable that it would be proved advisable to make various tests at some of the different factories. For all these reasons I think it would be highly inadvisable to have the different manufacturers directly represented. Not one of them, I believe, would be willing to give free access to its shops to a committee composed of members of rival companies.

THE PRESIDENT:—The matter that Dr. Hutchinson referred to, the composition of this committee—is a most important question, and it would be well, I think, if this matter were discussed as to whether manufacturers should or should not be actual members of that body.

I will call on Mr. Lieb for his views on this subject.

MR. JOHN W. LIEB, JR.:—I fear that I cannot add much to what has already been said on a rather broad subject. As to the desirability of standardizing types of apparatus, I am inclined to agree with what several of the previous speakers have said. It has been pointed out that the electrical industry bears the stamp of the general tendency of American manufacturing methods—construction, so as to secure interchangeability and duplication of parts. I think that the electrical industry, as of more recent development, has profited perhaps more largely than other industries by American manufacturing methods by using labor saving devices, working by jigs, templates and other devices for rapid duplication of parts. This is somewhat in contrast with European methods in similar industries, where, I think, a marked tendency is manifest to depart from standard types. This is the case, for instance, with direct driven dynamos, particularly if they are driven by turbines. Each turbine, practically, is designed and developed for the specific case, considering particularly the conditions of fall, discharge, etc. The result is, of course, the most efficient type of apparatus to fill each condition. It must, however, be admitted that it is rather an expensive method, each construction requiring special patterns and special development. I believe that the INSTITUTE would be doing valuable work for the industry generally, for the manufacturer and for the purchaser alike, if it could take hold of the work, and starting with certain definitions of quantity, follow on to define

or recommend what should be reasonably expected from a standard piece of apparatus as regards capacity, insulation and other characteristics that have already been touched upon by previous speakers.

I would like to point out one, I might almost say insignificant, phase of this question; its relation to electricity supply companies. The tendency to-day is away from the practice that obtained at one time of basing the charge for current delivered to electric motors, on the capacity of the motor, or the charge of a flat rate per month per motor, based on its horse-power. While we have gone beyond that stage of the industry, it is still the practice of many companies to consider the rated capacity of the motor as a factor in its system of charges for current, fixing a minimum charge per month, depending upon the size of the motor installed; the horse-power capacity of the installation, its equivalent in kilowatts or 16 candle-power lamps entering as a factor in fixing the rate. I will not go too far afield in this direction, as it is comparatively one of the unimportant elements. The committee having in hand such an important piece of work should, as suggested by one of the previous speakers, be untrammelled by any direct relations with the manufacturing companies. I am sure the manufacturing companies would be most happy to place at the disposal of the committee, all the information and practical data which they have at command, but it seems to me it would be undesirable for any manufacturer to be represented on the committee. I should think they would prefer not to be directly represented. On the other hand, it would certainly be desirable that the committee should profit by consulting with those who represent the different classes of service to which the apparatus is put. It might be desirable to have apparatus which is used in an isolated plant conform to certain conditions of overload and regulation, while apparatus in a large central station should satisfy other conditions; the committee will probably find it difficult to formulate any general rules or specifications covering all conditions of service, but they would certainly do very excellent work by setting up some standard for general reference in order to minimize the questions which are constantly arising along the lines of the discussion this evening.

THE PRESIDENT:—I think Mr. Lieb is right in saying that the manufacturers would probably prefer not to be on that committee; but it is open to question, I think, whether if it were composed entirely of purchasers of the apparatus it would be as competent a committee as if it were composed entirely of manufacturers. There are three sides to the question, as Mr. Rice has pointed out in his paper—the manufacturer, the purchaser and the consulting engineer, and leaving out any one of them you do not necessarily produce any better result. I would like to have Dr. Kennelly speak on that point. In private conversation he has taken a different position and I would like to have that side of it expressed.

DR. KENNELLY:—The attitude taken by the different speakers this evening upon that point as to how the committee should be made up, is somewhat different from the manner in which I have thought about it. It seemed to me that having a committee to recommend how manufacturers should make apparatus, without having any manufacturers on that committee, was something like playing Hamlet with Hamlet left out. This is a matter which vitally interests all manufacturers. It is for their benefit, as well as for the benefit of those who purchase from the manufacturers, that such a committee would ultimately be doing this work, and in some way or other the manufacturer must be consulted, whether they are consulted by having them off the committee and asking them what they think, or whether they are consulted by having them on such a committee and assisting the lucubrations of the committee with their actual presence, is a question which admits of several considerations. I think there is one advantage of having manufacturers on the committee—it would be a safeguard against anything being recommended by the committee which could possibly give umbrage to other manufacturers, because I think that if one manufacturer saw a recommendation going into the report of the committee that would look like unfair play, he would be the very person to object to it, and I think it would be a good safeguard against any injudicious recommendations being formulated. But I think that this is a matter of opinion, because sooner or later the manufacturers must be consulted.

THE PRESIDENT:—I will call upon Mr. Dunn to discuss this question.

MR. GANO S. DUNN:—The problem as it has been discussed increases so greatly in magnitude that I fear our action will be discouraged. Let us not attempt to deal with the whole matter or to cover the whole field at once. I recommend the appointment of a permanent committee that shall sit through several years, and from time to time, as convenient subjects come up, shall standardize and dispose of them. For instance, the methods of measuring temperature rise could be dealt with immediately.

At present three standards of temperature increase are in vogue, and while believed by many engineers to be nearly equivalent, are different to a great degree. One requires that the rise of temperature in no part of a machine shall exceed the specification limit. Another, that the rise of temperature as measured by thermometer shall not exceed the same limit, and another that the rise of temperature as measured by increase of resistance shall not exceed the limit.

I have in my hands details of certain experiments which I will not now give, because I see the discussion has been trending in another direction, but the results show temperature measurements by thermometer differing in some cases from temperature meas-

urements by resistance, by over 100 per cent., while in other cases the two methods reach a result different by only 20 per cent.

An immediate action that the committee could take would be to declare that in the opinion of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, the most accurate method of measuring temperature increase is by increase of resistance, and that as this method gives results considerably higher than the thermometer, the limits of rise should be newly set to be eqivalent to what has been the standard under the latter method.

This would settle the controversies arising over specifications where no method of measurement is specified, and would also direct attention to what the investigations of the INSTITUTE would have determined to be the most accurate method.

With regard to the composition of the committee, I think it would be very wrong to omit the manufacturers; as wrong as to omit the consulting engineers. I am not able to propose a solution, but my objection would be overcome by the appointment of members who are not manufacturers provided they had been at some time. The experience derived from actual contact with factory methods, is absolutely necessary to the successful determination of the questions affecting standardization of apparatus.

MR. H. B. COHO:—It seems to me that the proper way to do is to have two committees; not two committees in the INSTITUTE, but one a committee of the INSTITUTE, and the other a committee of manufacturers formed outside to consider their present standards. Of course, all manufacturers have their present standards. On the other hand, the committee of the INSTITUTE could, as Dr. Hutchinson states, suggest something that the manufacturers might work up to. Of course, if a manufacturer does not have to make new patterns, he considers himself so much better off, and naturally will not make new patterns if he can help it, whereas the committee from the INSTITUTE might suggest to the manufacturer improvements which he could from time to time adopt. There is one point, it seems to me, on which the INSTITUTE could do something, and that is in the matter of alternating current, especially with reference to motors. The great number of frequencies we have to contend with makes it almost impossible for a man to sell a motor of his own make to go on anything but his own generator. It is very seldom that a motor of one system will work on that of another, satisfactorily. Of course, I realize that this is one of the arguments used to sell complete plants of one make.

It seems to me a very good idea to have this matter taken up, but I agree with Dr. Hutchinson that manufacturers are hardly the proper persons to place on the committee, unless there is a representative from each of the manufacturers, it being, in my mind, very difficult to satisfy one manufacturer that another manufacturer is not prejudiced in his own favor.

I think the subject of alternating current, especially in regard to transformers and motors, would be a matter which could be readily brought up and discussed with a great deal of advantage.

THE PRESIDENT:—I doubt very much if the action taken will be far enough. The conservative idea that seems to prevail will prevent, I think, any action being taken which will discriminate between manufacturers. I think the action taken will be so very general that it will not fall hard on any individual. The idea at present seems to be that only a very general action should be taken, and I think it would not affect one manufacturer much more than another. It is simply the manufacturer's point of view in general, I think, that Mr. Dunn has contended for rather than any particular manufacturer's point of view.

MR. FRANK A. PATTISON:—It seems to me that we should consider why this question is brought before the INSTITUTE. There is evidently some cause that has made this question what we might call in electrical circles the living question of the day, and it seems to me it is due to the fact that we cannot understand each other. The manufacturer does not understand the engineer. The owner does not understand the engineer. The owner does not understand the manufacturer. Now, when two men start out in a discussion and want to arrive at some definite conclusion, the first thing that they reasonably agree upon is certain definitions which, if used, mean certain things and cannot mean anything else. Now all the disagreements that have come within the scope of my experience have arisen from the fact that people did not understand each other, simply because they did not understand the terms used in the same way. Therefore, it would appear that the solution of the problem is not in the standardization of the apparatus, which is an impossibility. Standardization of apparatus is a natural growth, and the minute you attempt to arbitrarily standardize anything that is going through a natural growth, you at once stint it, and it seems to me far from the right path of this INSTITUTE to attempt in any way to standardize the apparatus itself, which practically means that a 50-kilowatt machine shall be a 50-kilowatt machine "INSTITUTE standard" for example, that that means one thing, and that it cannot mean anything else; that whenever that term is used it must always mean the same thing. That is an impossibility. Standardization cannot be attained in that way. As to the committee which should take this question up, it seems to me that in a body of the standing of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS that they have no need to fear that the action of any of their committees will be questioned on account of the composition of that committee. Therefore, I am of the opinion most decidedly that that committee should be appointed by the Chair and should be composed of men who in the Chairman's opinion are best suited, regardless of whether they are manufacturers, engineers or users, and that that committee should be a

permanent committee; that that committee should define terms to be used, but should not attempt in any way to define or standardize apparatus at first, and that it should deal not only with the questions that are before us to-day, but that any questions wherein definitions arise that need standardization should come within the scope of that committee, and that it should be a permanent committee of the INSTITUTE.

MR. LOZIER:—My thoughts are very much in line with what Mr. Pattison says. I would like very much to get the sense of the meeting as to whether they think the standards should be passed only on safety limitations instead of what would be desirable performance of the apparatus. I think in a great many insurance standards laid down for boiler testing, and also for steam engine testing, the terminations there are on certain fixed lines which do not necessarily call for desirable performance, but that in fixing the lines on which the tests should be made, the apparatus is restricted to certain safety factors. Now if we could determine for motors, for example, (or for dynamos) what was an exact safety limit for a 10-hour operation, that safety limit could be modified for different types and forms of machines. This is a point on which I should like to have an expression of opinion.

THE PRESIDENT:—I do not understand exactly the points you desire to bring out.

MR. LOZIER:—The point is this: Cannot the INSTITUTE lay down certain safety limits? It would seem to me that for instance the question of sparking—of course, this is rather hard to determine—but take the question of temperature alone. That has a certain safety limit that I think the INSTITUTE could determine. Of course, there are other determinations that I think would appear desirable, but at the moment I have more particularly in mind the question of temperature, and if it would not be best to have those safety limits determined by such a committee rather than what they would consider to be a desirable temperature. One man, for example, might think it would be desirable to have 35 degrees C. for a 10-hour run, while another man would want 38, while the safety limit could be used as a fixed quantity.

THE PRESIDENT:—Do I understand that you desire to have an expression of the sense of the meeting as to the actual safety limit, as to the desirability of specifying a safety limit for a rise of temperature?

MR. LOZIER:—Yes, that general feature. I think that would give us one of the quantities that should be specified within the limits of safety. I do not think that limit has ever been positively determined. We have arrived at 40 degrees C. That seems to be the generally accepted safety limit. That might be so, but it might be less than that. It might be greater. But it seems to me that we could arrive at certain fixed constants that

would represent elements determined by the safety factor in distinction to what would appear to be desirable performance. I think "desirable features" is an open question.

I think this general plan of safety limits can be made to apply to more than the temperature, but I simply state that as an instance.

THE PRESIDENT:—Before putting this or any other question to vote, I think it would be well for the discussion to be completed, and it would also be well, I think, for the INSTITUTE to decide whether it cares to vote on these specific questions or not. Any question on the subject is in order.

MR. F. V. HENSHAW:—It seems to me before we go into these specific details as to what is to be standardized, and so on, it would be rather well to decide as to whether we will adopt the suggestion of a standing committee to take up these matters. I would like to say one thing in regard to the *personnel* of that committee. I do not quite understand why the INSTITUTE is so rough on the manufacturers. The standards which we have, and we have quite a number, as the result of a good many years of evolution, have all been developed by the manufacturers endeavoring to meet the demands of the public, and really to-day the best practical data we can get anywhere is from the published catalogues and tables of the various manufacturers. I furthermore agree with the recent speaker that members of the INSTITUTE ought to be like Cæsar's wife, above suspicion, in regard to furthering their own interests, and furthermore these recommendations will be submitted to the whole INSTITUTE, undoubtedly, before they are finally authorized by that body, and any member, if he has anything to object to, is always at liberty to object to it.

MR. N. M. HOPKINS:—I would like very much to hear a general opinion, not on the advisability of appointing a committee composed of manufacturing and consulting engineers, but on the probable work and decision of such a committee. I think a number of gentlemen have disfavored the appointing of a composite committee, so to speak, without thoroughly considering the question of their probable work and findings. I do not think there would be much conflict of ideas and opinions, even with such questions as efficiency of apparatus, highest permissible heating of armatures, field coils, etc. I do not think the findings of two separate committees, one composed of manufacturing engineers and the other of consulting or designing engineers would differ very materially. I do not think the manufacturing talent should be excluded from the committee.

DR. HUTCHINSON:—I see I have stirred up the meeting by my references to the electrical companies being represented on this committee. I gave my reasons for this position, and am sure that in practice the matter would turn out as I have stated. It would be necessary to have all the companies represented, or

none. If you have all represented, then the factories of all would be closed to the committee for any testing work that the committee might have to do. I believe that the committee can get all the information it asks from the companies without having one of their representatives on the committee.

MR. CHARLES P. STEINMETZ:—The problem before the house to-night is undoubtedly a very important and a very difficult one. Now, as regards the question of manufacturers, there are undoubtedly some very good reasons why manufacturers should not be on the committee, and there are some very good reasons also why they should be on the committee. But it appears to me there are still very much better reasons why this question should not have been raised at all. If the INSTITUTE intends to produce something of lasting value, which will be accepted and adopted by the whole continent, then the committee doing the work must be composed of men of such standing and reputation that, regardless of whether they are connected with manufacturing concerns or not, there can be no question that they will be impartial and not influenced by the fact that they are connected with this or that company. But they must be the best men available, and then there will be no question about it whether manufacturers or not. Besides, the problem under discussion appears to me to have two different phases: one is the question of standardizing the meaning of terms, and the other is the question of standardizing the methods of tests. To arrive at a common understanding of the meaning of terms, or the constants represented by them, it is undoubtedly extremely desirable to have the terms standards. I may only mention the unfortunate term "efficiency," which used to mean in the old times the output divided by output plus $C^2R$, because $C^2R$ was the only thing all people knew how to measure. But gradually it dawned upon them that there are some losses in the iron and consequently the question arose, should they be taken in the efficiency or not. After that there were other losses discovered in the brushes of the continuous current machine, etc. It is very desirable to standardize what efficiency means. Most of the discrepancies between engineers arise from the divergence in the understanding of the meaning of one of these terms. It may be quite desirable also to have standard methods of testing which is now one very great difficulty. The comparison with mechanical engineering is very nice, but we must consider that of the whole field of engineering, no branch is so much developed theoretically as electrical engineering, and if we consider the exactness of electrical engineering where you fight about a fraction of a per cent. of efficiency, with the exactness of mechanical engineering, where you make the beam that has to carry the structure of a bridge 500 per cent. stronger than calculated, you see there is great difference, and thus it may be quite desirable when making mechanical tests with the inaccuracy of mechanical engineering, to pro-

duce a standard or rather a recipe—how to make the test. But that would not do in electrical engineering. It would not do, for instance, in measuring efficiency, to say efficiency shall be tested by measuring the mechanical output by a brake, and the electrical output by an instrument. If you take a direct connected alternator of some 2,000-kilowatts capacity, you cannot get a mechanical brake to measure the input, and you cannot measure the output either, very well. From indicator diagrams of the engine you cannot get the efficiency, since the engine efficiency is so low that you cannot get any accuracy. So you see the only possibility in very many cases in determining these efficiencies is some abbreviated method. Efficiency usually is determined by measuring the losses and summing them up. Now, that is the best that can be done in many cases. But if the INSTITUTE should settle that efficiency shall be measured by adding certain losses it may be a dangerous thing, because there may be other losses coming in and there may be a difference of opinion as to whether there is loss or not. I mention, for instance, hysteresis loss, because hysteresis has always been my hobby. It is not quite agreed yet amongst all the engineers what should be charged to an alternator as hysteresis loss, whether the loss corresponding to the terminal voltage or the loss at a voltage corresponding to full load excitation at open circuit or some intermediary value. I especially gave attention to this feature and found that the nearest approach to correctness is derived by charging as loss the hysteresis loss corresponding to terminal voltage plus $CR$. It seems to me there may be other points of a similar nature where it would be liable for the INSTITUTE to settle on something that is wrong. So you see we might possibly discuss methods of testing before the INSTITUTE, or any committee, and make some recommendations, but we cannot with certainty determine these methods of tests as we should determine the meaning of terms, and that I think would be the most important work to be taken up by the committee at first namely, to decide on the meaning of terms used in electrical engineering.

MR. C. O. MAILLOUX:—I rise to a point of order and information. I think that the present Committee on Units and Standards is competent and that it is within its province to take up questions of terminology.

THE PRESIDENT:—Mr. Mailloux brings up the point that we have a standing Committee on Units and Standards, the name of which and the history of which would, I think, justify his position that the mere question of considering terms and definitions would be taken up by that committee and such questions have been considered by it in the past. This was considered to be rather a new and broader question than the one now proposed. It should be remembered that this committee would, in any case, report back to the INSTITUTE for final action. Therefore, the work of this committee would simply be preliminary and subject to revision.

MR. MAX OSTERBERG :—It seems to me that if we should have a committee which would determine on the general requirements of all machinery as regards temperature, different kinds of efficiency, etc., if we had a committee to determine these points, we would necessarily require a definition on the part of such a committee of what they mean by the individual terms. How, for example, they would recommend that temperature should be measured in order to satisfy the condition of what they call the requirement. It does not fully bind all the manufacturers to fulfil those, but it would certainly be one method to go by for the manufacturers in general and for the guidance of the consumer as to what he ought to require, or what he might be entitled to get. So it seems to me that since we have a Committee on Units and Standards, without treading on their toes in any way, we could have a committee to determine what the requirements of machinery should be and in that way get a direct distinct definition of the terms.

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After further discussion as to the propriety of appointing a committee to consider the question, the point was raised that no quorum was present. It was then voted upon motion of Mr. Hamblet, that the whole matter be referred to Council, after which the meeting adjourned.

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## Discussion at Chicago, January 26, 1898.

The Chairman, [Mr. B. J. Arnold]:—Prof. Stine has read to you the letters which have been sent on here from various members of the Institute and others in New York, calling our attention to the desirability of an action or expression on the part of the American Institute of Electrical Engineers regarding the standardization of electrical machinery. This is something which those of us who are in practical work are ready to consider and see if some standard can be adopted. The electrical manufacturers of to-day, in fact, are endeavoring to find out from engineers and purchasers of electrical machinery what their ideas and desires are as to the heating limits of the machinery. In fact, one of the largest manufacturers of electrical machinery has sent a representative throughout the country, calling upon the principal engineers, and getting their ideas as to what they thought these limits should be, and I have no doubt but that others are doing the same thing. Therefore there seems to be a great desire on the part of the manufacturers and certainly on the part of the purchasers and engineers to have some such standard which will be recognized, *as a standard*.

Mr. Meyer:—I think that a standard in any line of machines is certainly a very necessary thing. For instance, in machine shop practice and in electrical apparatus as well, there must be some uniformity. In dynamos it is a good thing to have all shafts of the same size and diameter, which could be used in different machines. In case of a breakdown, if there is but one shaft, it could only be used for the machine for which it was originally made. If any standard is made, it would be of great benefit. The necessity of standardizing this particular line has been recognized by many who use the apparatus and by the makers as well, for the latter have to carry a larger stock and invest heavily in machines to make apparatus, as well as in ready made parts, to supply orders on short notice. Outside of dynamos, I should think some standardization would be necessary, as well as instruments not so essential as dynamos, because instruments are usually less expensive than large dynamos. For arc lamps or incandescent lamps I should think there would not be so much standardizing required, for the lamps are now usually of the same voltage and candle power.

Mr. J. R. Cravath:—As I understand the subject, it is to consider the subject of standardizing the rating of apparatus, not the form of the apparatus itself. We all realize that some standards are needed. The only question is, what standards? It does not make a great deal of difference what standards are adopted, so that it is perfectly understood what those standards are. At present we are all at sea as regards to what the heating limit of a machine should be, and how it should be measured, and if we can come to some conclusion or arrive at some proper

definition as to what these heating limits are, we have made a great step in advance. The manufacturing concerns want it, I know. We need a standard now perhaps more in the railway field than in any other as is shown by the great frequency with which some manufacturers of electric railway motors change their methods of nomenclature in the attempt to get some way of rating more satisfactory to maker and customer. If however the INSTITUTE can formulate some standards for all to go by, the difficulty immediately disappears for it is only because of a lack of understanding as to how ratings are to be made that these differences exist.

MR. HICKOX:—Taking up the points for discussion in the order in which they are outlined on the black-board, let us first consider the proper and safe heating limits for dynamos and motors. For a machine working under normal condition, the temperature rise above the surrounding air should never exceed 80° Fah. on the armature core, 100° Fah. on the commutator, and 75° Fah. on the field coils, these temperatures being taken by applying thermometers to the outside of these parts. Standard specifications should give these heating limits after a continuous run of ten hours at the rated capacity of the machine.

Requirements for the sparkless range of the average machine should be that it should stand a change of load from no load to full load without any sparking at the brushes, they being fixed at the same position throughout the entire range.

Various engineers have their hobby as to what they consider the amount of overload a machine should stand safely, but I think that any dynamo or motor should be capable of being operated continuously for three hours at 25% overload, or for one hour at 50% overload, without causing any injurious effect.

In regard to the question of power rating, it is generally understood that dynamos should always be rated in kilowatts, and never in horse power, although the latter term does occasionally creep in.

Motors should always be rated in horse power, as that seems to be the most natural term for mechanical power, and the common formulas for work to be performed have H. P. for the unknown quantity.

If specifications are to definitely state what the insulation resistance of any machine shall be, I should think that a requirement of not less than one megohm would be sufficiently rigid for the average 110,220, or 500-volt dynamo or motor.

MR GEORGE A. DAMON:—That standard requirements for the manufacture of electrical apparatus would be desirable, is very well brought out by the tests which we recently made upon two 100 K. W. multipolar generators of the most modern types, but of different makes. One was a belted machine running at 600 turns per minute; the other was direct connected to a high speed engine. The belted machine after a run of six hours under full

load had a temperature in no part higher than 104°, less than 30° above that of the surrounding atmosphere, while on the other hand the direct connected generator showed a rise in temperature of over 90° after it had been running for the same length of time. One dynamo was certainly capable of standing an overload of 25 per cent. for a number of hours, while the other generator was working at about its safe limit. Both machines were accepted, but it is plain that one purchaser got a far better machine than did the other.

In regard to the heating:—The specification which a few years ago placed a limit of 75° Fah. rise in temperature upon generators was considered severe, now it is ordinary practice, and I believe we have reached a point where we may ask the manufacturers to be liberal with their ratings. The overload requirements influence, of course, the permissible heating limits, and should be standardized as far as possible at the same time. The engineer, who recently specified a machine of a certain capacity which "should be capable of standing a continuous overload of 50 per cent." at least did something to help along this movement.

Standard specifications should not only state the requirements, but should accurately describe the tests to which it is proposed to subject the apparatus in order to ascertain whether or not it fulfils the terms of the specifications. It is fully as important a matter to modify the methods of making tests upon electrical apparatus, as it is to standardize the requirements under which it is manufactured. It is not an unusual thing for engineers to prepare long and elaborate specifications for electrical machinery and then turn around and accept it after only a few crude tests or perhaps none at all. Take the matter of efficiencies for instance. We like to write in our specifications, 94½%, 94% and 93% as the requirements for the commercial efficiency of our generators at full, three-quarter, and half load, but how many of us are ever quite sure that we get what we call for? I find the literature upon commercial efficiency tests very incomplete. Efficiency curves are published now and then by manufacturers and others, but seldom are they accompanied by any indication of the method by which they were obtained. I find that even large manufacturers are dissatisfied with the accuracy of the results of their efficiency tests, and there seems to be no unanimity of opinion in regard to the best method to be used. Besides a few laboratory methods which are not commercially practicable, we have the Hopkinson method, the stray power method, the indicator method, and now comes a new method called the Routin method. But we know most of these methods are based more or less upon certain assumptions and are therefore liable to be incorrect. In the Hopkinson method we assume that the motor and generator have the same efficiency, whereas to operate the generator at full load it is necessary to overload the other machine

which is running as a motor. In the stray power method we assume that the loss in the armature when the machine is running itself as a motor with no load, is the same as when the generator is operating fully loaded, while in the indicator method we must necessarily assume that the loss by friction in both the engine and generator, as obtained in a rather uncertain way from the friction cards, remains constant at all loads. The question is, how accurate are these various methods and how will results by the different methods compare? It strikes me that a series of tests upon the same machines, under identical conditions, by the various methods, would be of value. Our Englewood and Chicago plant and the Chicago Board of Trade plant are admirably adapted for this purpose. In these plants the engines and generators are connected by means of the "Arnold system" so that it would be an easy matter to disconnect the generators from the engines and run any one as a motor, either independently or connected to another machine. As both plants are provided with storage batteries it would be comparatively easy to measure the power required to make up the losses in the manner provided for in either the Hopkinson, stray power, or Routin methods, and I hope to be able to obtain some data along this line. One of our prominent technical schools has its campus lighting plant connected by means of the "Arnold system" and a large builder of dynamo electric machinery is planning a testing table along the same lines, so that it is possible to obtain accurate information as to the relative limitations and possibilities of the various commercial systems of testing. It would be well then to accompany our standard specification efficiencies with a detailed account of at least one of the most reliable methods, if any such are found, and the same can be said of the tests which should be provided for the heating limits and the sparking, insulation and overload requirements.

PROF. STINE:—The proposition that there should be an attempt made to establish certain standards governing the drafting of specifications for the construction and operation of electrical apparatus, is meeting with such general favor that it argues a decided need for such things. The manufacturing companies have long felt that something should be done towards freeing them from burdensome, and, at times, unjust or impossible requirements which consulting engineers have imposed upon them. Recently, a set of specifications for some transformers was sent to certain manufacturers. The guarantee which they demanded from competing firms covering regulation, core aging, efficiency and temperature of operation, were incapable of being met in the present state of art, and plainly showed that the engineer who drafted them was not experimentally familiar with the subject. It is manifestly unjust to ask for bids on such a basis.

The desirability for standards governing construction, operation and testing of electrical apparatus is so generally admitted

that it scarcely requires further argument. It is rather a question of agreement on a certain number of points to make a beginning. The National Electrical Code, recently adopted, shows us how to go about the matter. When first framed, it was exceedingly crude and incomplete; but anything was sufficient for a beginning, and the elaboration of the rules, as time progressed and experience widened, has been most satisfactory.

In a similar spirit let a beginning be made towards standard specifications. We know a great deal more about this subject than the insurance people did about installation in the early days of the code. We are, then, better prepared for the task and will run less chance of making errors.

No association is so representative of all interests, or contains such a wealth of experience, ability and data as the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS. The question of immediate ways and means may be left with the Council, which in turn would doubtless approach the subject through an appropriate committee.

The functions of the consulting engineer are interpreted with the widest latitude. In many instances the consulting engineer attempts the duties and responsibilities of the designing engineer. But it is not always possible to separate these. The consulting engineer may properly specify both materials and design when he endeavors to make some particular combination, and the company receiving the order is so conservative as to be behind the times. But the consulting engineer is rarely in position to specify design and materials. He has had comparatively little to do with the progress in electrical apparatus. This has come principally from the manufacturing companies, and some few testing laboratories working in co-operation with them.

As matters now stand, the consulting engineer should draw up a detailed plan of what he wishes to accomplish. This should be submitted to the manufacturing companies as the basis for bids. They in turn should then fully specify the design, construction, materials and operation of the apparatus which they tender and state the guarantees. The engineer may then gather from the competing specifications and bids the possibilities upon which he may form a final set of specifications. This ensures the competing companies the full use of their experience in construction, selection of materials and data from tests.

At the present time, there is opportunity for only a limited number of standards which shall govern the consulting engineer and manufacturer in the preliminary work.

It would be manifestly unjust and a prevention of progress to adopt standards for induction value in the iron circuits of dynamos and motors, and the cores of transformers; to specify that cast iron shall be used here, and sheet steel there, etc. Such matters as these, and practically all details of design, construction and selection of materials should be left open.

But there is still an opportunity for many standard requirements; these occur in (a) the operation; (b) the testing; and (c) the rating of apparatus. Covering the tests of apparatus I wish especially to speak. Specifications usually make quite a matter of temperature limits for different portions of electrical apparatus. The assignment of such limits would be very properly within the duties of such a committee on standards. But the manner of testing whether the machine fulfils such requirements ought to be clearly prescribed. Every one who has carefully tested the heating of transformers under load knows that a number of different temperatures may be measured about the coils and core, depending upon the manner of applying the thermometer and the part where it is inserted. Here is a real necessity for a standard direction governing the use of a thermometer in ascertaining the temperature of cores, fields, windings, and bearing and conducting surfaces.

The efficiences of apparatus at various loads is usually strongly insisted upon in specifications. Yet those who devote most attention to such tests know that with the exception of photometric tests they are the least satisfactory of all electrical measurements, on account of the uncertainties in the estimation of the power applied to a given translating device. Though specifications often designate efficiencies to a second decimal, they are rarely measured within one or two per centum, at best, and more rarely are engineers able to verify their own specifications regarding efficiency. It is very desirable that a committee on standards should formulate precise directions for a determination of efficiencies.

So far as dynamo and motor characteristics enter in specifications, their determination should also be specified. In the same connection, the rating of the output of continuous and alternating current generators needs careful definition.

Another subject of great importance is the specification of the quality of insulating materials. Common practice states this in megohms, presumably to be determined by some galvanometer method for measuring high resistances. This is unjust to the progressive manufacturer. Such ratings are practically of little value as a guide to the operation of the apparatus. No manufacturer is justified in relying on a high insulation resistance, and specifications requiring this often place the most excellent apparatus at a disadvantage. The only adequate test of an insulation is its dielectric strength. As this is usually tested with a step-up transformer, there should be some standard agreement governing the periodicity and perhaps the wave form of the E.M.F. impressed on the insulation; also an accepted agreement making mere high resistance requirements secondary to those of dielectric strength.

MR. HICKOK:—I would like to say a word in regard to the insulation and the breakdown test. I think we should also include

a definite insulation resistance, for the reason that I have in mind a certain instance of a machine having its field coils wound with tape, and although it was submitted to the breakdown test, it had no injurious effect upon the machine, yet its measured insulation resistance was very low.

The customary rating of arc dynamos is in so many lamps, that is, 50, 75, 100, 125, etc. It might be better to rate them in watts.

PROF. STINE:—Do you consider it possible to-day to specify the efficiency of an arc machine.

MR. HICKOK:—Yes, it is possible to measure it within a fraction of one per cent. with a considerable degree of accuracy, but such an accurate test is necessarily an expensive one.

PROF. STINE:—Is it possible to come to any uniformity among makers on the question of efficiency of arc machines?

MR. HICKOK:—Most makers guarantee their machines, and they are frequently subjected to tests. I have several instances in mind of tests carefully made that gave very satisfactory results.

MR. ARNOLD:—If there is nothing further—no more questions to be asked, I would like to make a few statements as to what practice is followed in my office. On the question of heating limits of generators we have recently specified for some 500 kilowatt generators for light and power work, to be operated at 500 volts, 450 volts, 525 volts, for a certain station being built in this country, which is somewhat of a novelty, since the voltage is double what it ordinarily is in a direct current station. There are 440 lamps for the outside mains and 220 in the others. These generators have been specified pretty generally.

The sparking limits are such that the machines are practically sparkless all the way from no load to 50 per cent. overload, without changing the brushes. We expect to get that machine out. In fact, it is gotten out by a number of manufacturers to-day. I have seen the generator manufactured by the company Mr. Damon speaks of, stand the load from 0 to 50 per cent. overload without sparking. It is specified that the generators shall not flash when the circuit breaker is opened suddenly to 50 per cent. overload.

PROF. STINE:—In that same connection, do you put in any specification as to the self-induction of the armature?

MR. ARNOLD:—No. I am after the efficiency under different conditions of load. When we come to writing a practical specification, we have no means of determining that very closely.

These generators that I have mentioned are not to exceed these limits when operating under the following conditions at 25 per cent. overload for two hours, $33\frac{1}{3}$ per cent. overload for one hour, or 50 per cent. overload momentarily. The efficiencies we have required are as follows:

| | |
|---|---|
| At one quarter load | 90 per cent. |
| At one-half load | $93\frac{1}{2}$ " " |
| At three-quarter load | $94\frac{1}{2}$ " " |
| At overload | 95 " " |

Such a scheme should define the limits that the generators may have, and let those limits be far enough from the drooping point on the curve so that it has a somewhat surplus path to go over the rating. The sincere manufacturers to-day are coming close to these figures which I have given here. But as we all know, the capacity of a machine, its kilowatt output capacity depends entirely upon its heating limits. It is a matter of limiting the temperature to determine the capacity of the machine.

PROF. STINE:—Would you say that that would be the only limit or would you find a droop limit?

MR. ARNOLD:—There would be a slight variation of voltage when we get over the drooping point on the curve. As to how much that variation would be, I am not just at present prepared to say. I think it advisable to make such a limitation in specifications, or rather, a set of recommendations.

I also agree with one of the speakers to the effect that it is not advisable to specify sizes at all; but it is hardly within the scope of this discussion to go into the matter of sizes, as I understand it, because we wish to leave the matter of sizes to the manufacturer. Let him accomplish the result, but give us the thing we ask for in efficiency, heating limits and capacity.

PROF. STINE:—In specifications, I would specify weights, areas, and velocities, but I would not attempt to specify diameters, lengths or strengths. Possible the latter might be considered, but not the other two.

MR. ARNOLD:—Another thing: The only correct way to get the temperature of the field is to measure after or during the test. It is seldom done, but I believe it should be done, instead of trying to use a thermometer when we only get an approximation. Power rating should by all means be given in kilowatts, if possible. That is the only way to do it. We certainly have got the generators in shape for that basis, and in time we will measure engines in kilowatts instead of horse-power. In measuring generators by horse-power it has to be figured up in kilowatts afterward.

In our specifications we specify mica insulation, and seldom use anything else. For a generator in which anything but mica is used, the regulations depend upon the kind of work you want the machine to do. In the machine I have mentioned the winding should maintain the voltage at 500 without the assistance of the series winding.

PROF. STINE:—You mean to say up to full load? With what range in the rheostat?

MR. ARNOLD:—The rheostat controls from 400 to 500 volts. I do think this question is one of the most important which has come before the INSTITUTE since my membership in it, and it ought to be taken hold of in some kind of intelligent manner, and see if some result cannot be obtained which will bring the purchaser, the engineer and the manufacturer on some basis

where they can agree, and not have the constant argument which we have to-day as to how a machine ought to be rated. If there was a plan which, in the combined judgment of the best thought in this line in the country, it would relieve a great many of the embarrassing conditions which now exist.

MR. HICKOK:—There is always a question, when the subject of efficiency comes up, as to just what the term "efficiency" means. Some engineers include with the engine losses the entire friction of the revolving parts, and also the friction of the brushes on the commutator, thus giving to the dynamo an apparently high efficiency. I think specifications should define more particularly what is meant by this word.

[Adjourned.]

[Communicated after Adjournment by Charles F. Scott.]

The desirability of uniformity in ratings and specifications of electrical apparatus to consumer, consulting engineer and manufacturer, would be obvious to anyone, but their full advantage is best appreciated by those who suffer from the difficulties and confusion which now exist.

As an example, the rating of an alternating current generator may be mentioned. Suppose a generator of 100 k. w. at 1,000 volts is to be used for operating motors which have a power factor of 90 per cent., a "100-k. w." generator may mean:

(1) One which can deliver 100 k. w. to non-inductive circuits, such as for incandescent lighting.

(2) One which can deliver 100 amperes at 1,000 volts at a power factor of 90 per cent. The true output in power is only 900 k. w., while the tax upon the machine is considerably greater for delivering this current at 1,000 volts to an inductive load, as the field current must be notably increased on account of the reaction on the field caused by the lagging armature current.

(3) One which can deliver 100 k. w. of true energy to circuits having a 90 per cent. power factor. The current required is 111 amperes, and as the power factor is 90 per cent., the tax upon the machine is considerably greater than it would be if this current were delivered to a non-inductive load.

These three performances require machines differing in capacity and represented by machines which can give approximately 100, 110 and 120 k. w. to non-inductive circuits.

The fact that these characteristics of alternators are not generally and correctly understood, makes the matter of interpretation of indefinite specifications all the more difficult.

While it is desirable, it is not as essential to have absolute scientific accuracy in all of the definitions as it is to have definite definitions of capacity and performance, and definite methods of testing which are mutually understood.

As a representative of a manufacturing concern, I urge that the Institute carry to completion the good work which it has begun.

Pittsburgh, April 2nd, 1898.

# THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York, February 23d, 1898.

The 122nd meeting of the INSTITUTE was held this date at 12 West 31st Street, and was called to order by President Crocker at 8:20 P. M.

The Secretary announced that at the meeting of the Executive Committee in the afternoon the following associate members were elected:

| Name. | Address. | Endorsed by |
|---|---|---|
| BROILI, FRANK | Electrical Engineer, California Elec. Works; residence, 823 Geary St., San Francisco, Cal. | W. F. C. Hasson. Wynn Meredith. K. G. Dunn. |
| LIBBY, SAMUEL BYINGTON | Supt. N. Y. & S. I. Electric Co., West New Brighton, N. Y. | Chas. J. Bogue. W. G. Whitmore. Ralph W. Pope. |
| LOUIS, OTTO T | Manager of New York Branch, Queen & Co. Inc.: residence, 840 East 119th St., New York City. | Wm. A. Anthony. James Hamblet. Samuel Sheldon. |
| MORTLAND, JAMES A. | Prof. of Physics, Faculty State Normal School, 2502 Walnut St., Cedar Falls, Iowa. | Ralph W. Pope. Wm. J. Hammer. Wm. Maver, Jr. |
| SCHUM, CHAS. H. | Electrical Engineer, Ideal Electric Corp., 216 Third Ave., New York City. | F. A. LaRoche. R. W. Pope. H. J. Ryan. |
| SEDGWICK, C. E. | Agent at San Francisco Office, General Electric Co., 15 First St., residence, Berkeley, Cal. | J. A. Lighthipe. Wynn Meredith. F. F. Barbour. |

TRANSFERRED FROM ASSOCIATE TO FULL MEMBERSHIP.

Approved by Board of Examiners, Jan. 12th, 1898.

| | |
|---|---|
| GOLTZ, WILLIAM | Milwaukee, Wis. |

THE PRESIDENT:—The next business is the paper of this evening, "Single Phase Induction Motor," and I will call upon the author, Mr. Charles Steinmetz, to present it.

Mr. Steinmetz read the following paper:

*A paper presented at the 122d Meeting of the American Institute of Electrical Engineers, New York, February 23d, 1898, President Crocker in the Chair.*

# SINGLE PHASE INDUCTION MOTOR.

BY CHARLES PROTEUS STEINMETZ.

## §. 1. Load and Speed Curves of Single Phase Induction Motor.

In the polyphase motor a number of secondary coils displaced in position from each other, are acted upon by a number of primary coils displaced in position and excited by E. M. F.'s displaced in phase from each other by the same angle as the displacement of position of the coils.

In the single phase induction motor a system of armature circuits is acted upon by one primary coil (or a system of primary coils connected in series or in parallel) excited by a single alternating current.

A number of secondary circuits displaced in position must be used so as to offer to the primary circuit a short-circuited secondary in any position of the armature. If only one secondary coil is used, the motor is a synchronous induction motor and belongs to a different class, the reaction machines.

A single-phase induction motor will not start from rest, but when started in either direction will accelerate with increasing torque and approach synchronism.

When running at or very near synchronism, the magnetic field of the single-phase induction motor is identical with that of the polyphase motor. That is, the magnetic field has the same intensity in every direction, but is displaced in phase progressively, and can be represented by the theory of the rotating field. Thus in a turn wound at right angles to the primary winding of the single-phase induction motor, at synchronism an E. M. F. is in-

duced equal to that induced in a turn of the primary winding, but differing therefrom by 90° in phase.

In a polyphase motor the magnetic flux in any direction is due to the resultant M. M. F. of primary and of secondary currents, in the same way as in a transformer. The same is the case in the direction of the axis of the exciting coil of the single phase induction motor. In the direction at right angles to the axis of the exciting coil, however, the magnetic flux is due to the M. M. F. of the armature currents alone, no primary E. M. F. acting in this direction.

Consequently, while in the polyphase motor running light, that is doing no work whatever, the armature becomes current-less, and the primary currents are the exciting current of the motor only, in the single-phase induction motor, even when running light, the armature still carries the exciting current of the magnetic flux in quadrature with the axis of the primary exciting coil. Since this flux has the same intensity as the flux in the direction of the axis of the primary exciting coil, the current in the armature of the single-phase induction motor running light, and therefore also the primary current corresponding thereto, has the same M. M. F., that is, the same intensity as the primary exciting current, and the total primary current of the single-phase induction motor running light is thus twice the exciting current; that is, it is the exciting current of the main magnetic flux plus the current inducing in the armature the exciting current of the cross magnetic flux. In the armature or secondary, this exciting current is a current of twice the primary frequency.

Thus, if in a quarter-phase motor running light, one phase is open circuited, the current in the other phase doubles. If in the three-phase motor two phases are open circuited, the current in the third phase trebles, since the resultant M. M. F. of a three-phase machine is 1.5 times that of one phase. In consequence hereof, the total volt-ampere input of the motor remains the same, and at the same magnetic density, or the same impressed E. M. F., all induction motors, single-phase as well as polyphase, consume approximately the same volt-ampere input, and the same power input for excitation, and give the same distribution of magnetic flux.

Since the maximum output of a single-phase motor at the same impressed E. M. F. is considerably less than that of a polyphase

motor, it follows herefrom that the relative exciting current in the single-phase motor must be larger.

The cause of this cross magnetization in the single-phase induction motor near synchronism is that the induced armature currents lag 90° behind the induced magnetism, and are carried by the synchronous rotation 90° in space before reaching their maximum, thus give the same magnetic effect as a quarter-phase E. M. F. impressed upon the primary system in quadrature position with the main coil, and can be eliminated by impressing a magnetizing quadrature E. M. F. upon an auxiliary motor circuit as done in the monocyclic motor.

Below synchronism, the induced armature currents are carried less than 90°, and thus the cross-magnetization due to them is correspondingly reduced, and becomes zero at standstill.

The torque of an induction motor is proportional to the armature energy currents times the intensity of magnetic flux in quadrature position thereto. Thus in the polyphase motor, where the magnetic flux is constant in all directions at all speeds, and proportional to the counter E. M. F. $e$, the torque is

$$T = e\, I_1.$$

In the single-phase induction motor, the armature energy currents $I_1$ can flow only coaxial with the primary coil as the only position in which corresponding primary currents exist. The magnetic flux in quadrature position is proportional to the component $e$ carried in quadrature, or approximately to $(1-s)\, e$, and the torque is thus,

$$T = (1-s)\, e\, I_1$$

thus decreases much faster with decreasing speed, and becomes zero at standstill. The power is then,

$$P = (1-s)^2\, e\, I_1.$$

Since in the single-phase motor one primary only, but a multiplicity of secondary circuits exists, all secondary circuits are to be considered as corresponding to the same primary circuit, and thus as the secondary impedance is to be used the joint impedance of all secondary circuits. Thus, if the armature has a quarter-phase winding of impedance $Z_1$ per circuit, the resultant secondary impedance is $\frac{Z_1}{2}$, if it contains a three-phase winding of impedance $Z_1$ per circuit, the resultant secondary is $\frac{Z_1}{3}$.

In consequence hereof the resultant secondary impedance of a single-phase motor is less in comparison with the primary impedance than in the polyphase motor. Since the drop of speed under load depends upon the secondary resistance, in the single-phase induction motor the drop in speed at load is generally less than in the polyphase motor. This greater constancy of speed of the single-phase induction motor has led to the erroneous opinion that such a motor operates at synchronism, while in reality even at no load whatever it cannot approach perfect synchronism.

The further calculation of the single-phase induction motor is identical with that of the polyphase induction motor, as given in my former paper on the polyphase induction motor.[1]

In general, no special motors are used for single-phase circuits, but polyphase motors adapted thereto. An induction motor with one primary winding only, could not be started by a phase-splitting device, and would necessarily be started by external means. A polyphase motor, as for instance a standard three-phase motor operating single phase, by having two of its terminals connected to the single-phase mains, is just as satisfactory a single-phase motor as one built with one primary winding only. The only difference is that in the latter case a part of the circumference of the primary structure is left without winding, while in the polyphase motor this part contains windings also, which, however, are not used, or not effective when running as single-phase motor, but are necessary when starting by means of displaced E. M. F.'s. Thus in a three-phase motor operating from single phase mains, in starting, the third terminal is connected to a phase-displacing device, giving to the motor the cross-magnetization in quadrature to the axis of the primary coil, which at speed is produced by the rotation of the induced secondary currents, and which is necessary for producing the torque by its actions upon the induced secondary energy currents.

Thus the investigation of the single-phase induction motor resolves itself into the investigation of the quarter-phase or three-phase, or in general polyphase motor operating on single-phase circuits.

If in a quarter-phase motor, or motor with two primary circuits in quadrature,

$Y^1 = g^1 + jb^1 =$ primary exciting admittance per circuit,

1. TRANSACTIONS, vol. xiv, p. 175, (May issue)

the same motor operating as single-phase motor on one primary circuit has the primary admittance,

$$Y = 2\,Y^1 = g + jb.$$

Since the total volt-ampere and power input when running light are the same, as seen before.

In the same way in the three-phase motor, or motor with three primary circuits if,

$Y'' = g'' + jb'' =$ primary exciting admittance per circuit,

the same motor operated as single-phase motor has the primary admittance:

$$Y = 3\,Y'' = g + jb.$$

The primary impedance is that of the circuits used when running single phase.

The secondary impedance is the resultant impedance of all secondary circuits, reduced to the primary by the ratio of turns, since all secondary circuits correspond to one and the same primary circuit in the single-phase motor. Thus if $Z_1^1 = r_1^1 - j\,x_1^1 =$ secondary impedance per circuit of a motor with quarter-phase secondary, or secondary of two circuits in quadrature, the secondary impedance of the same motor on single-phase circuit is,

$$Z_1 = \tfrac{1}{2}\,Z_1^1 = r_1 - j\,x_1.$$

If $Z_1'' = r_1'' - j\,x_1'' =$ secondary impedance per circuit, of a motor with three-phase secondary reduced to the primary, the secondary impedance of the same motor on single phase circuit is,

$$Z_1 = \tfrac{1}{3}\,Z_1'' = r_1 - j\,x_1.$$

Or in general, in a motor with quarter-phase secondary, the secondary impedance is halved; in a motor with three-phase secondary, reduced to $\frac{1}{3}$, since the total secondary winding corresponds to one primary winding. Thus the slip is less in the single-phase motor.

In diagrams Figs. 1, 2, 3, 4 and 5, the load curves of five typical single phase induction motors are shown.

These motors are identical with the five typical polyphase motors shown as Figs. 3, 4, 5, 6, and 7 of my previous paper[1] on the polyphase induction motor, and their constants, are abstracted on Table I. of my previous paper, under $\times_1$, $\times_2$, $\times_3$, $\times_4$, and $\times_5$ on three-phase circuit, under $\times_1^1$, $\times_2^1$, $\times_3^1$, $\times_4^1$, $\times_5^1$ of this table as single-phase motor.

1. TRANSACTIONS, vol. xiv, p. 175, (May issue.)

These motors are:

| | Three-Phase, per circuit; | Single-Phase. |
|---|---|---|
| 1st—All around good motor: | | |
| | $Y = .01 + .1\,j$, | $Y = .03 + .3\,j$, |
| | $Z = .1 - .3\,j$, | $Z_0 = .1 - .3\,j$, |
| | or: | $Z_1 = .033 + .1\,j$, |
| | $\vartheta = 6.36$, | $\vartheta = 12.72$, |
| | $\beta = 10.0$ | $\beta = 10.0$, |
| | $\gamma = 31.6$, | $\gamma = 31.6$. |
| | Fig. 3. | Fig. 1. |
| 2nd.—High resistance motor: | | |
| | $Y = .01 + .1\,j$, | $Y = .03 + .3\,j$, |
| | $Z = .2 - .3\,j$, | $Z_0 = .2 - .3\,j$, |
| | or: | $Z_1 = .067 - .1\,j$, |
| | $\vartheta = 7.26$, | $\vartheta = 14.52$, |
| | $\beta = 10.0$, | $\beta = 10.0$, |
| | $\gamma = 55.4$, | $\gamma = 55.4$. |
| | Fig. 4. | Fig. 2. |
| 3rd.—High resistance and high admittance motor: | | |
| | $Y = .04 + .4\,j$, | $Y = .12 + 1.2\,j$, |
| | $Z = .3 - .3\,j$, | $Z_0 = .3 - .3\,j$, |
| | or: | $Z_1 = .1 - .1\,j$, |
| | $\vartheta = 34.2$, | $\vartheta = 68.4$, |
| | $\beta = 10.0$, | $\beta = 10.0$, |
| | $\gamma = 70.7$, | $\gamma = 70.7$. |
| | Fig. 5. | Fig. 3. |
| 4th.—High reactance motor: | | |
| $Y = .044 + 4\,j$, $Z = .05 - .3\,j$, | or $\begin{cases} Y = .02 + .2\,j, \\ Z = .1 - .6\,j, \end{cases}$ | $Y = .06 + .6\,j$, |
| | or: | $Z_0 = .1 - .6\,j$, |
| | | $Z_1 = .033 - .2\,j$, |
| | $\vartheta = 24.44$, | $\vartheta = 48.88$, |
| | $\beta = 10.0$, | $\beta = 10.0$, |
| | $\gamma = 16.4$, | $\gamma = 16.4$. |
| | Fig. 6. | Fig. 4. |
| 5th.—High susceptance motor: | | |
| | $Y = .02 + 4\,j$, | $Y = .06 + 1.2\,j$, |
| | $Z = .1 - .3\,j$. | $Z_0 = .1 - .3\,j$, |
| | or: | $Z_1 = .033 + .1\,j$, |
| | $\vartheta = 25.35$, | $\vartheta = 50.7$, |
| | $\beta = 5.0$, | $\beta = 5.0$, |
| | $\gamma = 31.6$, | $\gamma = 31.6$. |
| | Fig. 7. | Fig. 5. |

## TABLE I.

## CONSTANTS OF INDUCTION MOTORS.

$E_0 = 110.$

(The motors marked by × are typical motors.)

| No. of motor. | No. of diagram. | $Y = \frac{}{x+jb}$ | $Z = \frac{}{r-jx}$ | $\vartheta = \frac{}{2yz = y(z_0+z_1)}$ | $\beta = \frac{x}{y}$ | $\gamma = \frac{r}{z} = \frac{r_0+r_1}{z_0+z_1}$ | $T_m$ = max. torque, synchr. kilowatt. | $P_T$ = rated output, kw. at $\frac{1}{2}\sqrt{T_m}$. | $P_m$ = max. output, in per cent. of $P_T$. | $s$ = slip, at $P_T$ | Exciting current, per cent. of current at $P_T$ | Power Factor at | | | Efficiency at | | | Apparent efficiency at | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | full load. | ½ load. | ¼ load. | full load. | ½ load. | ¼ load | full load. | ½ load. | ¼ load. |
| | | .005 + .05 $j$ | .1 − .3 $j$ | 3.18 | 10.0 | 31.6 | 9.00 | 5.65 | 1.33 | 6.3 | 8.7 | 96.6 | 95 | 91 | 95 | 93 | 93.5 | 92.2 | 88 | 85 |
| | | .01 + .1 $j$ | .05 − .3 $j$ | 6.12 | 10.0 | 16.4 | 8.90 | 5.75 | 1.42 | 3.2 | 16.7 | 88.7 | 88.7 | 77.8 | 92.4 | 93.5 | 90.8 | 82 | 83.1 | 70.5 |
| ×1 | 3 | .01 + .1 $j$ | .1 − .3 $j$ | 6.36 | 10.0 | 31.6 | 8.25 | 5.15 | 1.36 | 6.3 | 17.4 | 89 | 88.5 | 74 | 86.5 | 91.2 | 88.2 | 77.4 | 81 | 65 |
| ×2 | 4 | .01 + .1 $j$ | .2 − .3 $j$ | 7.36 | 10.0 | 55.4 | 7.10 | 4.90 | 1.35 | 11 | 19.5 | 91 | 88.4 | 71 | 76 | 86.5 | 85 | 69.9 | 76 | 60 |
| | | .01 + .1 $j$ | .4 − .3 $j$ | 10.04 | 10.0 | 80.0 | 6.30 | 3.90 | 1.13 | 6 18 | 25.0 | 92 | 89 | 67 | 66 | 85 | 80 | 61 | 79 | 51 |
| | | .02 + .2 $j$ | .1 − .3 $j$ | 12.70 | 10.0 | 31.6 | 7.90 | 4.95 | 1.35 | 6.3 | 33 | 93 | 74 | 55 | 84 | 87 | 80 | 70 | 64.5 | 44 |
| | | .02 + .2 $j$ | .2 − .3 $j$ | 14.52 | 10.0 | 55.4 | 6.80 | 4.00 | 1.26 | 11 | 35 | 85 | 73 | 50.5 | 75.5 | 80.6 | 74 | 64.3 | 58 9 | 37.5 |
| ×4 | 6 | .04 + .4 $j$ | .05 − .3 $j$ | 24.44 | 10.0 | 16.4 | 7.90 | 5.10 | 1.42 | 3.2 | 51 4 | 69.4 | 55 | 37 | 86.4 | 88 | 73.5 | 60.5 | 45.5 | 27 |
| ×5 | 7 | .02 + .4 $j$ | .1 − .3 $j$ | 25.25 | 5.0 | 31.6 | 7.60 | 4.75 | 1.35 | 6.3 | 52 | 71 | 54 | 36 | 82 | 82.3 | 75 | 58.2 | 44.5 | 26 |
| | | .04 + .4 $j$ | .1 − .3 $j$ | 25.40 | 10.0 | 31.6 | 7.30 | 4.55 | 1.35 | 6.3 | 53.5 | 71 | 54 | 36 | 73.7 | 79 | 69 | 56 | 42.5 | 25 |
| ×3 | 5 | .04 + .4 $j$ | .3 − .3 $j$ | 34.20 | 10.0 | 70.7 | 5.50 | 3.10 | 1.13 | 16.5 | 62.5 | 75 | 58 | 37 | 60.6 | 61.3 | 47 | 45.3 | 32.5 | 17 |
| | | Single | phase | Motors: | | | | | | | | | | | | | | | | |
| $1_s$ | | .08 + .3 $j$ | .1 − .3 $j$ | 12.72 | 10 | 31.6 | 9.9 | 6.4 | 1.43 | 2.8 | 36 | 82 | 73 | 52.5 | 84.5 | 85 | 78 | 70 | 61.5 | 40.5 |
| | | | .033 − .1 $j$ | | | | | | | | | | | | | | | | | |
| $2_s$ | | .08 + .3 $j$ | .2 − .3 $j$ | 14.52 | 10 | 55.4 | 7.5 | 4.8 | 1.36 | 4.3 | 44.3 | 85 | 67.5 | 47 | 74.5 | 78 | 69 | 62 | 53 | 33.5 |
| | | | .067 − .1 $j$ | | | | | | | | | | | | | | | | | |
| $4_s$ | | .12 + 1.2 $j$ | .05 − .3 $j$ | 48.98 | 10 | 16.4 | 8.8 | 5.8 | 1.43 | 1.9 | 73 | 92 | 86 | 24.5 | 76 | 68 | 52 | 39.5 | 24.5 | 13 |
| | | | .0167 − .1 $j$ | | | | | | | | | | | | | | | | | |
| $5_s$ | | .06 + 1.2 $j$ | .1 − .3 $j$ | 50.7 | 5 | 31.6 | 7.75 | 5.0 | 1.41 | 3.4 | 78 | 53.5 | 35 | 24 | 69.5 | 62.5 | 47.5 | 37 | 22 | 11 |
| | | | .033 − .1 $j$ | | | | | | | | | | | | | | | | | |
| $3_s$ | | .12 + 1.2 $j$ | .3 − .3 $j$ | 68.4 | 10 | 70.7 | 4.3 | 2.67 | 1.33 | 6.8 | 92 | 60.5 | 45.7 | 28.5 | 40 | 28.6 | 17 | 24.3 | 13.5 | 6.8 |
| | | | .1 − .1 $j$ | | | | | | | | | | | | | | | | | |

That is, the three-phase motors $\times_1$, $\times_2$, $\times_3$, $\times_4$, $\times_5$ operated from a single-phase circuit give the constants shown under $\times_1^1$, $\times_2^1$, $\times_3^1$, $\times_4^1$, $\times_5^1$. The primary admittance is increased threefold, the primary impedance remains the same, and the secondary impedance is decreased to $\frac{1}{3}$.

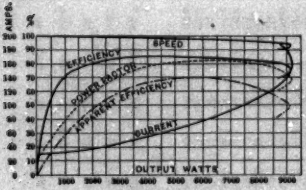


FIG. 1.—Single Phase Induction Motor. Load Curves.
$Y = .03 + .3j.$ $Z = .033 - .1j.$
$Z_0 = .1 - .3j.$

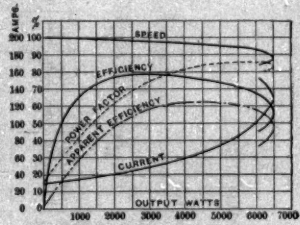


FIG. 2.—Single Phase Induction Motor. Load Curves.
$Y = .03 + .3j.$ $Z_1 = .067 - .1j.$
$Z_0 = .2 - .3j.$

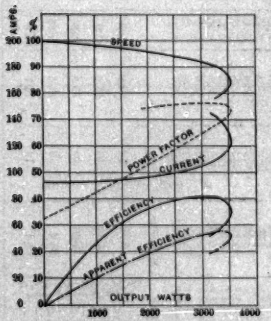


FIG. 3.—Single Phase Induction Motor. Load Curves.
$Y = .12 + 1.2j.$ $Z_1 = .1 - .1j.$
$Z_0 = .3 - .3j.$

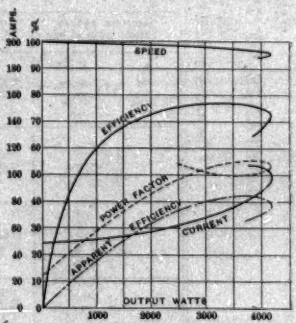


FIG. 4.—Single Phase Induction Motor. Load Curves.
$Y = .06 + 6j.$ $Z_1 = .033 - .2j.$
$Z_0 = .1 - .6j.$

Since in a single-phase motor the primary impedance and the secondary impedance are generally very different, the approximation $Z_0 = Z_1$ can no longer be made, and in deriving the

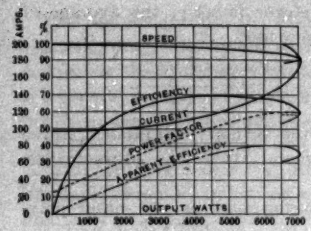


FIG. 5.—Single-Phase Induction Motor.
Load Curves.
$Y = .06 + 1.2j. \quad Z_1 = .033 - .1j.$
$Z_0 = .1 - .3j.$

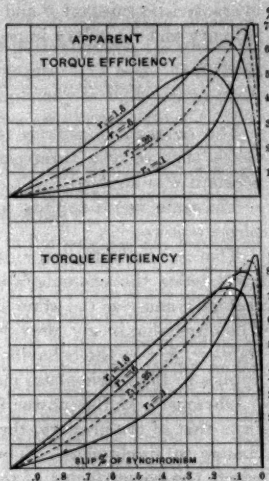


FIG. 7.—Single Phase Induction Motor.
$Y = .03 + .3j. \quad Z_1 = .033 - .1j.$
$Z_0 = .1 - .3j.$

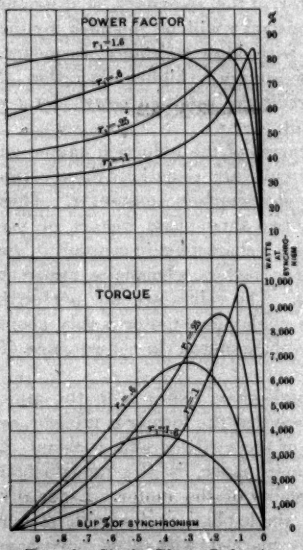


FIG. 6.—Single Phase Induction Motor.
$Y = .03 + .3j. \quad Z_1 = .033 - .1j.$
$Z_0 = .1 - .3j.$

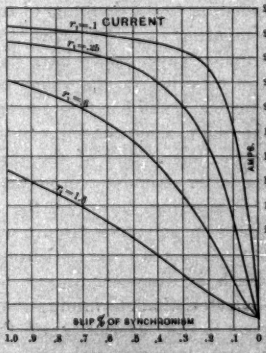


FIG. 8.—Single Phase Induction Motor.
$Y = .03 + .3j. \quad Z_1 = .033 - .1j.$
$Z_0 = .1 - .3j.$

characteristic constant $\vartheta$ and the power factor $\gamma$, the average of primary and secondary impedance is used.

$$Z = Z_0 + Z_1 = \frac{r_0 + r_1}{2} - j\ \frac{x_0 + x_1}{2}.$$

The average impedance is $\frac{2}{3}$, the primary admittance, three times that of the three-phase motor, thus the characteristic constant $\vartheta$ is doubled.

The same result applies to quarter-phase or any other polyphase motors.

That is: An induction motor operated on single-phase circuit gives twice the characteristic constant $\vartheta$, but the same power factors $\beta$ and $\gamma$ as the same motor operated on polyphase circuit. The discussion in my previous paper on the effect of the constants $\vartheta$, $\beta$, $\gamma$ on the behavior of the polyphase motor essentially applies to the single-phase motor also.

Obviously, if the power factor of the secondary is different from that of the primary, the constant $\gamma$ will be modified by the change to single-phase. Thus with high resistance secondary, where the power factor of the secondary is higher than that of the primary, on single-phase circuit the resultant power factor $\gamma$ is lower than on polyphase circuit.

The differences in the load curves of the induction motor on single-phase and on polyphase circuit, are essentially those corresponding to the doubling of the characteristic constant $\vartheta$. Thus comparing $\times_1^1$, $\times_2^1$ with the two three-phase motors of my previous paper which have the same characteristic constant $\vartheta$, we find very nearly the same power factors, efficiencies, apparent efficiencies and exciting currents. The values are not perfectly the same, since the slip $s$ in the single phase motor is only about half or less of that of the three-phase motor, in consequence of the lesser secondary impedance.

Comparing, however, the single-phase motor with the same motor on polyphase circuit, we see that the former is inferior, as to be expected from the change of the characteristic constant $\vartheta$, which, as discussed in my former paper, is the fundamental characteristic determining the quality of the motor. Especially great is the inferiority of the single-phase motor with poorer motors; less, although still very marked with good polyphase motors. The single-phase motor is inferior, especially at light load, mostly due to its lower power factor.

The best insight into the changes which have taken place by operating polyphase motors on single-phase circuits, is given by comparing the load curves in the diagrams. As seen, the effect is a general decrease of the curves, especially at light loads, making them rise much slower and to lesser maximum values.

In general it can be said that a fair polyphase motor makes a poor single-phase motor. A good polyphase motor makes a fair single-phase motor, and to get a good single-phase motor an exceedingly good polyphase motor is required.

The speed is much more constant in the single-phase induction motor than in the polyphase motor.

The maximum torque and output of the single-phase motor is somewhat greater than that of the three-phase motor per circuit. Hence, since the total torque and output of the three-phase motor is three times that per circuit, the single-phase motor at the same impressed E. M. F. gives less than half the output of the polyphase motor. This is the cause of the inferiority of the single-phase motor. With the same volt-ampere excitation, the same loss of energy in the iron, and somewhat over half the loss of energy in the copper, the output is reduced to less than one-half.

Obviously by operating the single-phase motors at higher voltage, or what means the same, rewinding them for a higher density, their output can be increased, but as discussed before, their constants are not changed thereby. Limiting the output by the heating of the motor, that is the loss of energy therein, the permissible rating of a motor on single-phase circuit is about from $\frac{2}{3}$ to $\frac{3}{4}$ that of the same motor rewound for polyphase circuits. In this case, the magnetic density of the single-phase motor is from 25 to 30% higher: thus the exciting volt-amperes and the loss in the iron correspondingly higher, as represented by the constants of the motor.

The falling off of the torque and the output of the single phase motor with decreasing speed, is much more abrupt than in the polyphase motor, since the expression of the torque of the single-phase motor contains the speed $(1-s)$ as factor, while that of the polyphase motor does not.

Thus, while in the polyphase motor the insertion of resistance in the armature does not change the torque, but merely shifts all its values toward lower speeds, in the single-phase motor resistance in the armature not only shifts the values of torque towards

lower speed, but also reduces the values due to the factor $(1-s)$ entering the expression of torque. In Figs. 6, 7 and 8 are shown speed curves of the motor $\times_1{}^1$ for the armature resistances,

$r_1 = .033$ (short-circuited armature),
$r_1 = .0833$,
$r_1 = .2$,
$r_1 = .533$.

corresponding to the resistances .1, .25, .6 and 1.6 in each of the three armature circuits. Thus these figures give the torque, power factor, torque efficiency, apparent torque efficiency and current as function of the speed on single-phase circuits, of the same motor and for the same armature resistances, for which the curves on three-phase circuit are shown in Figs. 10 and 11 of my previous paper. Thus these diagrams are directly comparable.

The torque curves of the single-phase motor have the same general character as those of the three-phase motor; a stable branch between synchronism and maximum torque and an unstable branch between maximum torque and standstill. The difference, however, is that all the single-phase torque curves slope toward zero at standstill, while the three-phase torque curves reach definite values at standstill. With increase of armature resistance, the maximum torque point shifts towards lower speed, but at the same time decreases in the single-phase motor, while it remains the same in the three-phase motor. In the single-phase motor, the maximum torque point is nearer synchronism, that is, the speed regulation is better.

In the single-phase motor a stable and an unstable branch of the torque curve always exists. In the polyphase motor the latter disappears beyond a certain secondary resistance, and for instance no longer exists in the above discussed motor for $r_1 = .6$ and $r_1 = 1.6$.

The current curves in Figs. 7 and 8 on single-phase and on three-phase circuits show the same general character, with the modification due to the lesser slip of the single-phase motor, which causes the same shape to correspond to a higher resistance. For instance, $r_1 = 1.6$ in the single-phase motor gives about the same shape as $r_1 = .6$ in the three-phase motor. The power factor curves have the same character, but the single-phase curves reach a lower maximum only, at a lesser slip.

Torque efficiency and apparent torque efficiency are different in so far as these quantities in polyphase motors approach finite

values at standstill, while in single-phase motors their curves slope towards zero at standstill. The torque efficiency and apparent torque efficiency reach their maxima at higher speed or lesser slip on single-phase than on polyphase circuit, and are lower.

It is obvious herefrom that the single-phase induction motor, while quite fair in the range of speed above the maximum torque point, becomes very unsatisfactory below this point, or at lower speeds. Armature resistance in the single phase induction motor brings the maximum torque point down to lower speeds. But since the maximum torque point is lowered, the more the speed is reduced at which it takes place, single-phase motors cannot be operated on rheostatic control for variable speeds as satisfactorily as polyphase motors, and armature resistance is thus used mostly only for starting the motor.

Since the torque of the single-phase motor falls off in any case with decreasing speed, the use of variable armature resistance in starting a single-phase motor is far more necessary than in polyphase motor. For instance, the three-phase motor in Fig. 10 of my previous paper will carry one-third of full load from standstill up to synchronism with short-circuited low resistance armature, and practically full load from one-half speed. The same motor as single-phase motor will carry one-third load from one-fourth speed, or two-thirds load, from one-half speed up to synchronism, only when the armature resistance is the most favorable, $r_1 = .6$.

At standstill the single-phase motor torque is zero, and thus some starting device, electrical or otherwise, is needed. This starting device, however, while it needs not to bring the motor up to full synchronism, as necessary with a single-phase synchronous motor, has to bring it up to some speed, since only at about one-fourth speed the inherent torque of the single-phase induction motor becomes noticeable, and considerable only at about one-half speed. With short-circuited low resistance armature, the starting device has to bring the motor up fairly close to synchronism.

Concatenation or tandem control of single-phase induction motors can be used, but the result is not as satisfactory as with polyphase motors, since the magnetic field of the single-phase motor becomes uniform only when approaching synchronism, and thus at half speed the E.M.F. induced in the secondary is pulsating in intensity.

The effect of the change of frequency and change of the number of poles, etc. is the same in the single-phase motor as in the polyphase motor, and the investigation regarding hereto is directly applicable to polyphase as well as single-phase motors.

In the preceding discussion of the single-phase motor, the assumption has been made that the magnetic field of cross-magnetization produced by the armature currents is proportional to the counter E. M. F. of the motor and to the speed. In reality it varies with a more complex function of the speed, and thus the torque curves calculated with this assumption, do not show the perfect coincidence with the results of tests as is the case of the polyphase motor, although they agree fairly well.

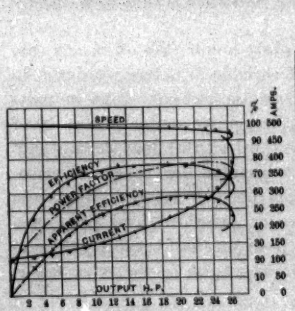


FIG. 9.
1.8-30-900-110, Form "A." 60 Cycles, 8 Poles, 110 volts. Single Phase.
Curves Calculated with $Y = 135 + 1.15\,j$.
$Z_0 = .045 - .124\,j$. $Z_1 = .0137 - .041\,j$.
Friction: 500 Watts at Synchronism.
Observed by Test: +

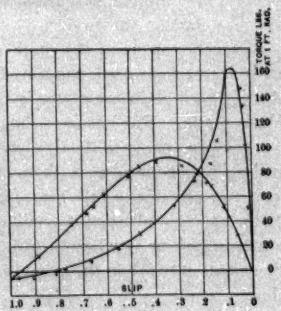


FIG. 10.
1.8-30-900-110, Form "A." 60 Cycles, 8 Poles, 110 Volts. Single Phase.
Speed Curves
Calculated with: $Y = .135 + 1.15\,j$.
$Z_0 = .045 - .124\,j$. $Z_1 = .0134 - .041\,j$.
500 Synchronous Watts Fricti n.
× Observed by Test.

A further assumption has been made by representing the M. M. F. of cross-magnetization by a doubling of the primary impedance, but neglecting the effect of the armature currents producing the cross-magnetization.

In Fig. 9 are shown the load curves, and in Fig. 10 the speed curves of the 1—8—30—900—1060 cycles, form A, the same motor of which the three-phase load curves are given in Fig. 1 and the three-phase speed curves in Fig 2 of my previous paper. These single-phase curves are calculated for the same impressed voltage, that is the same magnetic density, and with the same

constants as the three-phase curves in the previous paper, with the changes discussed above; trebling the primary admittance and reducing the secondary impedance to one-third. The results of tests are marked by crosses. As seen, the coincidence between test and calculation is quite good. This motor is the first 60-cycle induction motor built, and thus does not now represent the present state of the art, but is used here, owing to the very complete investigations made with it.

## § 2. Single-Phase Induction Motor Starting Devices.

As seen in the preceding, the single-phase induction motor at standstill has no starting torque, since the line of polarization due to the armature currents coincides with the axis of magnetic flux impressed by the primary circuit. Only when revolving, torque is produced, due to the axis of armature polarization being shifted against the axis of magnetism by the rotation, until at or near synchronism it is in quadrature therewith, and the magnetic disposition thus identical with that of the polyphase induction motor.

Leaving out of consideration starting by mechanical means, and starting by converting the motor into a series or shunt motor, that is by passing the alternating current by means of commutator and brushes through both elements of the motor, as methods of starting single-phase motors are left:

1st.—Shifting of the axis of armature or secondary polarization against the axis of inducing magnetism.

2nd.—Shifting the axis of magnetism, that is producing a magnetic flux displaced in position from the flux inducing the armature currents.

The first method requires a secondary system which is unsymmetrical in regard to the primary, and thus, since the secondary is movable, requires means of changing the secondary circuit, that is commutator brushes short-circuiting secondary coils in the position of effective torque, and open-circuiting them in the position of opposing torque.

Thus this method leads to the repulsion motor, which is a commutator motor also.

With the commutatorless induction motor, or motor with permanently closed armature circuits, all starting devices consist in establishing an auxiliary magnetic flux in phase with the induced

secondary currents in time, and in quadrature with the line of armature polarization in space, that is they consist in producing a component of magnetic flux in quadrature in space with the primary magnetic flux inducing the armature currents and in phase with the latter; that is in quadrature with the primary magnetic flux.

Thus if

$P$ = polarization due to the induced or armature currents,

$M$ = auxiliary magnetic flux,

$\varphi$ = phase displacement in time between $M$ and $P$, and

$\omega$ = phase displacement in space between $M$ and $P$, the torque is

$$T = P\,M \sin\,\omega \cos\,\varphi.$$

In general the starting torque, apparent torque efficiency, etc. of the single-phase induction motor with any of these devices are given in per cent. of the corresponding values of the same motor with polyphase magnetic flux, that is, with a magnetic system consisting of two equal magnetic fluxes in quadrature in time and space.

Thus all starting devices of the commutatorless single-phase induction motor consist in the production of a component of magnetic flux displaced from the axis of polarization of the induced or armature currents.

The infinite variety of arrangements proposed for this purpose can be grouped into three classes.

1st. *Phase-Splitting Devices.*—The primary system is composed of two or more circuits displaced from each other in position, and combined with impedances of different inductance factors so as to produce a phase displacement between them.

When using two motor circuits, they can either be connected in series between the single-phase mains, and shunted with impedances of different inductance factors, as for instance, a condensance and an inductance, or they can be connected in shunt between the single-phase mains but in series with impedances of different inductance factor. Obviously the impedance used for displacing the phase of the exciting coils can either be external or internal as represented by high-resistance winding in one coil, etc.

In this class belongs the use of the transformer as phase-splitting device, by inserting a transformer primary in series to one

motor circuit in the main line, and connecting the other motor circuit to the secondary of the transformer, or by feeding one of the motor circuits directly from the mains, and the other from the secondary of a transformer connected across the mains with its primary. In either case, it is the internal impedance, respectively internal admittance, of the transformer which is combined with one of the motor circuits for displacing its phase, and thus this arrangement becomes most effective by using transformers of high internal impedance or admittance, that is poor transformers.

2d. *Inductive Devices.*—The motor is excited by the combination of two or more circuits which are in inductive relation to each other. This mutual induction between the motor circuits can either take place outside of the motor in a separate phase-splitting device, or in the motor proper.

In the first case the simplest form is the divided circuit whose branches are inductively related to each other by passing around the same magnetic circuit external to the motor.

In the second case the simplest form is the combination of a primary exciting coil and a short-circuited secondary coil induced thereby on the primary member of the motor, or a secondary coil closed by an impedance.

3d. *Monocyclic Starting Device.*—An essentially wattless E. M. F. of displaced phase is produced outside of the motor, and used to energize a cross-magnetic circuit of the motor, either directly by a special teaser coil on the motor, or indirectly by combining this wattless E. M. F. with the main E. M. F. and thereby deriving a system of E. M. F.'s of approximately three-phase or any other relation. In this case the primary system of the motor is supplied essentially by a polyphase system of E. M. F. with a single-phase flow of energy, a system, which I have called "monocyclic."

## A. Phase Splitting Devices.

*Parallel Connection.* Let the motor contain two primary circuits of different inductance factors in parallel between the single-phase mains, and at right angles with each other in space.

Two equal primary circuits, of the apparent impedance,[1]

$$Z = r - j x,$$

1. Including secondary circuit, that is, $Z = \frac{\text{impressed E. M. F.}}{\text{current}}$ in motor coil

are arranged at right angles on the motor field, and connected in series with the respective impedances:

$$Z_1 = r_1 - j\,x_1 \text{ and } Z_2 = r_2 - j\,x_2$$

in parallel across the single-phase mains of impressed E. M. F.,

$$\mathbf{E} = e$$

as shown diagrammatically in Fig. 11.

Let $\mathbf{E}_1$ and $\mathbf{I}_1$ = E. M. F. and current in first,

$\mathbf{E}_2$ and $\mathbf{I}_2$ = " " " " second motor coil.

The angle of phase displacement between the E. M. F.'s, $\mathbf{E}_1$ and $\mathbf{E}_2$ be:

$$(\mathbf{E}_1, \mathbf{E}_2) = \omega.$$

The torque of the motor is proportional to:

$$T_s = E_1\,E_2 \sin \omega \qquad [1]$$

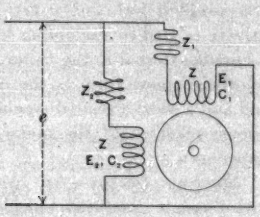


FIG. 11.

while the torque of the same motor with uniform or polyphase flux, that is as quarter-phase motor with E. M. F. $e$ per phase, is proportional to:

$$T_p = e^2.$$

Hence, the *relative starting torque* of the single-phase motor, or the ratio:

$\frac{\text{torque single-phase}}{\text{torque polyphase}}$ at same impressed E. M. F. $e$, is:

$$t = \frac{T_s}{T_p} = \frac{E_1\,E_2}{e^2} \sin \omega. \qquad (1)$$

Let: $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$ = total current taken by the motor on single phase circuit of E. M. F. $e$, and $I$ = absolute value thereof.

1. $\mathbf{E}_1$ $\mathbf{I}_1$ etc. denoting the complex vectors, $E_1$, $I_1$ their absolute or scalar values.

The total current taken by the motor on quarter-phase circuit of E. M. F. $e$ per phase is:

$$I_0 = \frac{2e}{\sqrt{r^2 + x^2}} \tag{2}$$

hence, the *ratio of currents*:

$$c = \frac{I}{I_0} = \frac{I\sqrt{r^2 + x^2}}{2e} \tag{3}$$

and consequently, the *ratio of apparent torque efficiencies* of the motor on single-phase and on polyphase circuits, or the *relative apparent torque efficiency* of the motor starting device, is:

$$\delta = \frac{t}{c} = \frac{T_s}{T_p} \times \frac{I_0}{I}$$
$$= \frac{2E_1 E_2 \sin \omega}{eI\sqrt{r^2 + x^2}}. \tag{4}$$

In this single-phase motor starting device, it is:

Current in first motor coil:

$$I_1 = \frac{e}{Z + Z_1}, \tag{5}$$

in the second motor coil:

$$I_2 = \frac{e}{Z + Z_2}, \tag{6}$$

hence, total current:

$$I = I_1 + I_2 = \frac{e\,(2\,Z + Z_1 + Z_2)}{(Z + Z_1)(Z + Z_2)}, \tag{7}$$

or, absolute:

$$I = \frac{2e\sqrt{\left(r + \frac{r_1 + r_2}{2}\right) + \left(x + \frac{x_1 + x_2}{2}\right)^2}}{\sqrt{[(r + r_1)^2 + (x + x_1)^2]\,[(r + r_2)^2 + (x + x_2)^2]}}, \tag{8}$$

and, ratio of currents:

$$c = \frac{I}{I_0} = \frac{\sqrt{r^2 + x^2}\sqrt{\left(r + \frac{r_1 + r_2}{2}\right)^2 + \left(x + \frac{x_1 + x_2}{2}\right)^2}}{\sqrt{(r + r_1)^2 + (x + x_1)^2}\,\sqrt{(r + r_2)^2 + (x + x_2)^2}}. \tag{9}$$

The E. M. F. across the first motor coil is:

$$E_1 = Z\,I_1 = \frac{e\,Z}{Z + Z_1},$$
$$= \frac{e\,(r - j\,x)}{(r + r_1) - j\,(x + x_1)}, \tag{10}$$

or absolute:

$$E_1 = \frac{e\sqrt{r^2+x^2}}{\sqrt{(r+r_1)^2+(x+x_1)^2}}, \tag{11}$$

and analogously, across the second motor coil:

$$\mathrm{E}_2 = \frac{e\,(r-jx)}{(r+r_2)-j\,(x+x_2)}, \tag{12}$$

$$E_2 = \frac{e\sqrt{r^2+x^2}}{\sqrt{(r+r_2)^2+(x+x_2)^2}} \tag{13}$$

The phase displacement $\omega$ between the E. M. F.'s $E_1$ and $E_2$ is:

$$a\,(\cos a + j\sin\omega) = \frac{\mathrm{E}_1}{\mathrm{E}_2},$$

$$= \frac{(r+r_2)-j\,(x+x_2)}{(r+r_1)-j\,(x+x_1)} = \tag{14}$$

$$\frac{[(r+r_1)(r+r_2)+(x+x_1)(x+x_2)]+j\,[r(x_1-x_2)+r_2(x+x_1)-r_1(x+x_2)]}{(r+r_1)^2+(x+x_1)^2},$$

hence:

$$a = \frac{\sqrt{(r+r_2)^2+(x+x_2)^2}}{\sqrt{(r+r_1)^2+(x+x_1)^2}},$$

and:

$$\sin\omega = \frac{r\,(x_1-x_2)+r_2\,(x+x_1)-r_1\,(x+x_2)}{\sqrt{(r+r_1)^2+(x+x_1)^2}\ \sqrt{(r+r_2)^2+(x+x_2)^2}}. \tag{15}$$

Substituting (11) (13) (15) into (1) gives the *relative torque of starting device*:

$$t = \frac{(r^2+x^2)\,[r\,(x_1-x_2)+r_2\,(x+x_1)-r_1\,(x+x_2)]}{[(r+r_1)^2+(x+x_1)^2]\,[(r+r_2)^2+(x+x_2)^2]}, \tag{16}$$

and, (9) and (16) combined, give the
*relative apparent torque efficiency of the starting device*:

$$\delta = \frac{t}{c} = \frac{\sqrt{r^2+x^2}\,[r(x_1-x_2)+r_2(x+x_1)-r_1(x+x_2)]}{\sqrt{(r+r_1)^2+(x+x_1)^2}\ \sqrt{(r+r_2)^2+(x+x_2)^2}\ \sqrt{(r+\frac{r_1+r_2}{2})^2+(x+\frac{x_1+x_2}{2})^2}} \tag{17}$$

## Special Cases.

1. *Resistance a in series with one motor circuit:*

$$r_2 = a, \qquad r_1 = x_1 = x_2 = 0.$$

Relative torque:

$$t = \frac{a\,x}{(r+a)^2+x^2}.$$

Relative current:

$$i = \frac{\sqrt{\left(r + \frac{a}{2}\right)^2 + x^2}}{\sqrt{(r+a)^2 + x^2}}$$

Relative apparent torque efficiency:

$$\delta = \frac{a\,x}{\sqrt{(r+a)^2 + x^2}\,\sqrt{\left(r + \frac{a}{2}\right)^2 + x^2}}.$$

2. *Reactance a in series with one motor circuit:*

$$x_1 = a, \qquad r_1 = r_2 = x_2 = 0.$$

Relative torque:

$$t = \frac{a\,r}{r^2 + (x+a)^2}.$$

Relative current:

$$i = \frac{\sqrt{r^2 + \left(x + \frac{a}{2}\right)^2}}{\sqrt{r^2 + (x+a)^2}}.$$

Relative apparent torque efficiency:

$$\delta = \frac{a\,r}{\sqrt{r^2 + (x+a)^2}\,\sqrt{r^2 + \left(x + \frac{a}{2}\right)^2}}.$$

Thus, reactance $a$ in series with one motor circuit gives the same values, but with $r$ and $x$ exchanged for each other, as resistance $a$ in one motor circuit, or: a reactance $a$ has on a motor of impedance $Z = b - jd$ the same effect as a resistance $a$ on a motor of impedance $Z = d - jb$.

In a motor of high reactance (permanently short-circuited armature) resistance $a$ in series, in a motor of high resistance (variable starting resistance) reactance $a$ in series gives better torque and apparent torque efficiency.

As instances are shown in Fig. 12, the relative apparent torque efficiency, or apparent torque efficiency $\frac{\text{single-phase}}{\text{polyphase}}$, as function of the reactance $a$ inserted in one of the two equal primary motor circuits, for the motors with apparent impedance:

1. $Z = 1 - 3\,j.$
2. $Z = 3 - 1\,j.$

If $a$ is resistance instead of reactance, the two curves in Fig. 12 merely exchange their position, as shown in parenthesis.

3. *Inductance and Capacity.*

$$x_2 = -(x_1 + 2x).$$

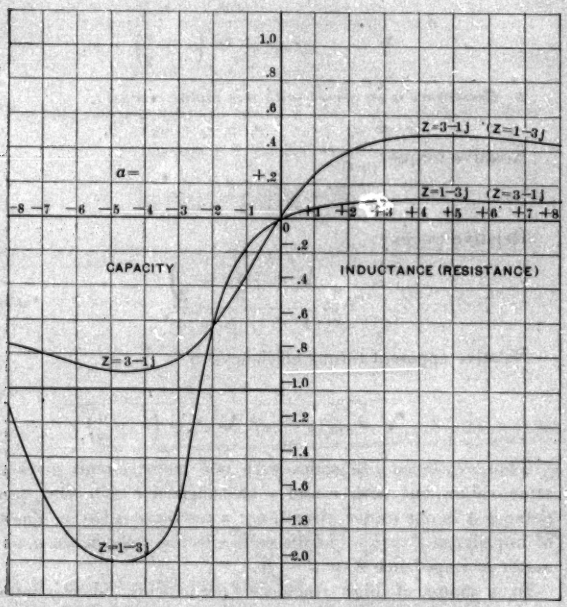


FIG. 12.

Single Phase Induction Motor. Starting Device.

Reactance $= \pm a$ in Circuit of one of the two Motor Primaries. Total Impedance per Motor Coil. $Z = 3 - 1j$ or $1 - 3j$.

A Phase Splitting Device.

Ratio of Apparent Torque Efficiencies. $\frac{\text{Single Phase.}}{\text{Polyphase.}}$

Relative torque:

$$t = \frac{2\,(r^2 + x^2)\,(x \times x_1)\left(r + \frac{r_1 + r_2}{2}\right)}{[(r + r_1)^2 + (x + x_1)^2]\;[(r + r_2)^2 + (x + x_1)^2]}.$$

Relative current:

$$i = \frac{\sqrt{r^2 + x^2}\left(r + \frac{r_1 + r_2}{2}\right)}{\sqrt{(r + r_1)^2 + (x + x_1)^2}\ \sqrt{(r + r_2)^2 + (x + x_1)^2}}$$

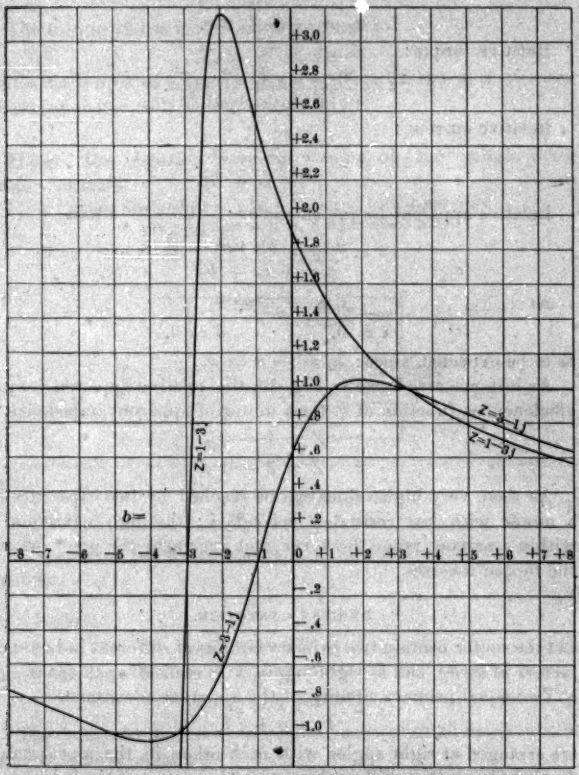


FIG. 13.—Single Phase Induction Motor. Starting Device.
Reactance $-jb$ and: $+j(b+2\times)$. In Circuit of the two Motor Coils of Total Impedance, $Z = r - jx = 3 - 1j$ or: $1 - 3j$.
Ratio of Apparent Torque Efficiencies. A Phase Splitting Device.

$$\frac{\text{Single Phase}}{\text{Polyphase}}$$

Relative apparent torque efficiency:

$$\delta = \frac{2\sqrt{r^2+x^2}\,(x+x_1)}{\sqrt{(r+r_1)^2+(x+x_1)^2}\;\sqrt{(r+r_2)^2+(x+x_1)^2}}.$$

3 *b*.—

$$x_1 = b, \qquad r_1 = r_2 = 0.$$
$$x_2 = -(b+2x.$$

Relative torque:

$$t = \frac{2\,r\,(x+b)\,(r^2+x^2)}{[r^2+(x+b)^2]^2}.$$

Relative current:

$$i = \frac{r\sqrt{r^2+x^2}}{r^2+(x+b)^2}.$$

Relative apparent torque efficiency:

$$\delta = \frac{2\,(x+b)\,\sqrt{r^2+x^2}}{r^2+(x+b)^2}.$$

At: $b = -x$:

$$t = 0, \qquad \delta = 0,$$

as to be expected, since: $x_2 = -x = x_1$.

As instance are shown in Fig. 13, the relative apparent torque efficiency, as function of $b$, for a motor of apparent impedance:

1. $Z = 1 - 3j$.
2. $Z = 3 - 1j$.

As seen, very high values can be reached by two capacities in a motor with low resistance secondary; the high maximum is within a narrow range, however, and very near the point where the torque reverses.

### Series Connection.

Let the motor contain two primary circuits of different inductance factors in series, and at right angles with each other in space.

Two equal primary circuits, of the apparent admittance[1]

$$Y = g + j\,b$$

are arranged at right angles with each other in the motor field, connected in series with each other in the circuit of the impressed E. M. F. $E = 2e$, and shunted by the respective admittances:

$$Y_1 = g_1 + j\,b_1 \text{ and } Y_2 = g_2 + j\,b_2,$$

as shown diagrammatically in Fig. 14.

Let:

$E_1$ and $I_1$ = E. M. F. and current in the first,

$E_2$ and $I_2$ = " " " " " second motor coil,

and:

$(E_1, E_2) = \omega$ the angle of phase displacement between $E_1$ and $E_2$.

The torque of the motor is proportional to:

$$T_s = E_1 E_2 \sin \omega \qquad [2]$$

while the torque of a quarter-phase motor with the E. M. F. $e$ per phase (or motor coil) is proportional to:

$$T_p = e^2.$$

Hence, the *relative starting torque* of the single-phase motor, or ratio.

$$\frac{\text{torque single-phase at E. M. F. } 2e \text{ across both motor coils,}}{\text{torque polyphase at E. M. F. } e \text{ per motor coil,}}$$

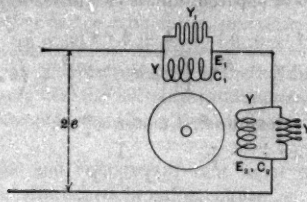


FIG. 14.

is:

$$t = \frac{T_s}{T_p} = \frac{E_1 E_2 \sin \omega}{e^2}. \qquad (1)$$

The current per motor coil at E. M. F. $e$ and quarter-phase circuit is:

$$I_0 = e\ Y. \qquad (2)$$

or, absolute:

$$I_0 = e\ \sqrt{b^2 + g^2}, \qquad (3)$$

hence the apparent input, quarter-phase:

$$2\ e\ I_0 = 2\ e^2 \sqrt{b^2 + g^2}. \qquad (4)$$

The joint admittance of the motor coils with their shunts, on single-phase circuits is:

$$\frac{1}{\frac{1}{Y + Y_1} + \frac{1}{Y + Y_2}} = \frac{(Y + Y_1)(Y + Y_2)}{2\ Y + Y_1 + Y_2},$$

1. and 2. see page 110.

hence the current,
at impressed E. M. F. $2e$:

$$I = \frac{e\,(Y + Y_1)\,(Y + Y_2)}{Y + \frac{Y_1 + Y_2}{2}}, \tag{5}$$

or absolute:

$$I = \frac{e\sqrt{(g + g_1)^2 + (b + b_1)^2}\ \sqrt{(g + g_2)^2 + (b + b_2)^2}}{\sqrt{\left(g + \frac{g_1 + g_2}{2}\right)^2 + \left(b + \frac{b_1 + b}{2}\right)^2}} \tag{6}$$

and, the apparent input:

$$2\,e\,I = \frac{2\,e^2\sqrt{(g + g_1)^2 + (b + b_1)^2}\ \sqrt{(g + g_2^2) + b + b_2)^2}}{\sqrt{\left(g + \frac{g_1 + g^2}{2}\right)^2 + \left(b + \frac{b_1 + b_2}{2}\right)^2}} \tag{7}$$

hence, the ratio of apparent inputs or *relative apparent input*:

$$e = \frac{2\,e\,I}{2\,e\,I_1} = \frac{\sqrt{(g + g_1)^2 + (b + b_1)^2}\sqrt{(g + g_2)^2 + (b + b_2)^2}}{\sqrt{g^2 + b^2}\sqrt{\left(g + \frac{g_1 + g_2}{2}\right)^2 + \left(b + \frac{b_1 + b_2}{2}\right)^2}} \tag{8}$$

The E. M. F. across the first motor coil is:

$$E_1 = \frac{I}{Y + Y_1}; \tag{9}$$

hence, substituting (5) in (9):

$$E_1 = \frac{e\,(Y + Y_2)}{Y + \frac{Y_1 + Y_2}{2}}, \tag{10}$$

or, absolute:

$$E_1 = \frac{e\sqrt{(g + g_2)^2 + (b + b_2)^2}}{\sqrt{\left(g + \frac{g_1 + g_2}{2}\right)^2 + \left(b + \frac{b_1 + b_2}{2}\right)^2}} \tag{11}$$

and analogously, $E_2$.

It is:

$$E_2 = \frac{I}{Y + Y_2}.$$

This combined with (9) gives the angle $\omega$ between $E_1$ and $E_2$ as:

$$a\,(\cos\omega + j\sin\omega) = \frac{Y + Y_2}{Y + Y_1} = \frac{(g + g_2) + j\,(b + b_2)}{(g + g_1) + j\,(b + b_1)}, \tag{12}$$

and herefrom, analogously as on page 122:

$$\sin \omega = \frac{g\,(b_1 - b_2) + g_2\,(b + b_1) - g_1\,(b + b_2)}{\sqrt{(g+g_1)^2 + (b+b_1)^2}\ \sqrt{(g+g_2)^2 + (b+b_2)^2}} \quad (13)$$

(11) and (13) substituted in (1) gives the *relative torque:*

$$t = \frac{g\,(b_2 - b_3) + g_2\,(b + b_1) - g_1\,(b + b_3)}{\left(g + \frac{g_1 + g_2}{2}\right)^2 + \left(b + \frac{b_1 + b_2}{2}\right)^2}, \quad (14)$$

and, (14) combined with (8) gives the *relative apparent torque efficiency:*

$$\delta = \frac{t}{c} =$$

$$\frac{\sqrt{g^2+b^2}\,[g\,(b_1 - b_2) + g_2\,(b + b_1) - g_1\,(b + b_2)]}{\sqrt{(g+g_1)^2+(b+b_1)^2}\ \sqrt{(g+g_2)^2+(b+b_2)^2}\ \sqrt{\left(g+\frac{g_1+g_2}{2}\right)^2+\left(b+\frac{b_1+b_2}{2}\right)^2}} \quad (15)$$

As seen, the relative apparent torque efficiency of the single-phase motor starting device with series connection leads to the same expression in the admittances, as parallel connection leads in the impedances. That is, replacing in the formula for parallel connection, on page 122, the resistances and reactances by the conductances and susceptances, give the formula for series connection.

Relative torque and relative apparent input would give the same formulas in admittances in series connection, if in this case the values for constant impressed primary current had been compared with the values for constant impressed primary E. M. F. in parallel connection. The apparent torque efficiency is independent of E. M. F. or current, thus gives the same expression.

## Special Cases.

1. *Conductance (or resistance) a shunting one motor coil.*

$$g_2 = a, \quad g_1 = b_1 = b_2 = 0.$$

Torque:

$$t = \frac{a\,b}{\left(g + \frac{a}{2}\right)^2 + b^2}.$$

Current:

$$i = \frac{\sqrt{(g+a)^2 + b^2}}{\sqrt{\left(g + \frac{a}{2}\right)^2 + b^2}}.$$

App. Torque Eff.

$$\delta = \frac{a\,b}{\sqrt{\left(g+a\right)^2+b^2}\sqrt{\left(g+\frac{a}{2}\right)^2+b^2}}$$

2. *Susceptance (or reactance) a shunting one motor coil.*

$$b_1 = a, \quad g_1 = g_2 = b_2 = 0.$$

Torque:

$$t = \frac{a\,g}{g^2+\left(b+\frac{a}{2}\right)^2}.$$

Current:

$$i = \frac{\sqrt{g^2+(b+a)^2}}{\sqrt{g^2+\left(b+\frac{a}{2}\right)^2}}.$$

App. Torque Eff.

$$\delta = \frac{a\,g}{\sqrt{g^2+\left(b+a\right)^2}\sqrt{g^2+\left(b+\frac{a}{2}\right)^2}}.$$

Obviously, regarding the effect and the relative advantages of conductance or susceptance, the same applies as stated on p. 4, regarding resistance and reactance.

Instance:

$$\alpha. \quad Y = 1 + 3\,j$$
$$\beta. \quad Y = 3 + 1\,j.$$

3. *Inductance and capacity.*

$$b_1 = a,$$
$$b_2 = -(a + 2\,b),$$
$$g_1 = g_2 = 0.$$

Torque:

$$t = \frac{2\,(a+b)}{g}.$$

Current:

$$i = \frac{g^2+(a+b)^2}{g\sqrt{g^2+b^2}}.$$

App. Torque. Eff:

$$\delta = \frac{2\,(a+b)\sqrt{g^2+b^2}}{g^2+(a+b)^2}$$

## B. Inductive Devices.

### External inductive devices.

*Inductively divided circuit.*—In its simplest form, as shown in diagram Fig. 15, the motor contains two circuits at right angles, of the same admittance:

$$Y = g + jb.$$

The one circuit (1) is in series with the one, the other (2) with the other of two coils wound on the same magnetic circuit $M$.

Let:

$e$ = E. M. F. across single-phase mains.

$Z = -j\,x$ = impedance of one turn on the magnetic circuit $M$.

$n_1$, $I_1$, $E_1$ = number of turns of $M$, current and E. M. F. across motor coil of circuit (1); $n_2$, $I_2$, $E_2$ the corresponding values of circuit (2).

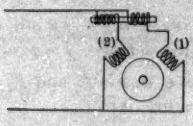


Fig. 15.

If both coils on $M$ are wound in the same direction, $n_1$ and $n_2$ are positive; if the coils are wound in opposite direction, $n_1$ and $n_2$ are of opposite signs.

The M. M. F. acting upon magnetic circuit $M$ is:

$$f = n_1 I_1 + n_2 I_2, \tag{1}$$

consequently, the E. M. F. induced per turn on $M$:

$$f Z = (n_1 I_1 + n_2 I_2) Z, \tag{2}$$

hence the E. M. F. across coil (1) on $M$:

$$E_1' = n_1 f Z = n_1 (n_1 I_1 + n_2 I_1)\, Z, \tag{3}$$

and across coil (2):

$$E_2' = n_2 f Z = n_2 (n_1 I_1 + n_2 I_2)\, Z. \tag{4}$$

The currents in the motor circuits (1) and (2) are:

$$I_1 = E_1\, Y, \tag{5}$$

$$I_2 = E_2\, Y. \tag{6}$$

Substituting (5) and (6) in (3) and (4), it is:

$$E_1^1 = n_1 (n_1 E_1 + n_2 E_2) Z Y, \tag{7}$$

$$E_2^1 = n_2 (n_1 E_1 + n_2 E_2) Z Y. \tag{8}$$

It is, however:

$$E_1 + E_1^1 = e = E_2 + E_2^1. \tag{9}$$

Thus, substituting (7) and (8):

$$E_1 (1 + n_1^2 Z Y) + n_1 n_2 E_2 Z Y = e, \tag{10}$$

$$n_1 n_2 E_1 Z Y + (1 + n_2^2 Z Y) E_2 = e, \tag{11}$$

hence:

$$E_1 = e \frac{1 + n_2 (n_2 - n_1) Z Y}{1 + (n_2^2 + n_1^2) Z Y}, \tag{12}$$

$$E_2 = e \frac{1 - n_1 (n_2 - n_1) Z Y}{1 + (n_2^2 + n_1^2) Z Y}, \tag{13}$$

or substituting $Z = -j x$; $Y = g + j b$:

$$E_1 = e \frac{1 + x b n_2 (n_2 - n_1) - j x g n_2 (n_2 - n_1)}{1 + x b (n_2^2 + n_1^2) - j x g (n_2^2 + n_1^2)}, \tag{14}$$

$$E_2 = e \frac{1 - x b n_1 (n_2 - n_1) + j x g n_1 (n_2 - n_1)}{1 + x b (n_2^2 - n_1^2) - j x g (n_2^2 + n_1^2)}. \tag{15}$$

The condition that the E. M. F.'s $E_1$ and $E_2$, and thus the two motor fluxes, are in quadrature, is:

$$E_2 = j a E_1, \tag{16}$$

where:

$a$ = ratio of E. M. F.'s $\frac{E_2}{E_1}$, and thus of fluxes produced by them.

Substituting (14) and (15) in (16), it is:

$$1 - x b n_1 (n_2 - n_2) + j x g n_1 (n_2 - n_1) = j a + j a x b n_2 (n_2 - n_1) + a x g n_2 (n_2 - n_1).$$

thus:

$$x b n_1 (n_2 - n_1) + a x g n_2 (n_1 - n_1) = 1,$$

$$x g n_1 (n_2 - n_1) - a x b n_2 (n_2 - n_1) + a,$$

or:

$$x (n_2 - n_1) (b n_1 + a g n_2) = 1, \tag{17}$$

$$x (n_2 - n_1) (g n_1 - a b n_2) = a, \tag{18}$$

hence:

$$\frac{b n_1 + a g n_2}{g n_1 - a b n_2} = \frac{1}{a}, \tag{19}$$

herefrom:

$$a = \frac{-b\,(n_1 + n_2) \pm \sqrt{b_2\,(n_1 + n_2)^2 + 4\,g^2\,n_1\,n_2}}{2\,n_2\,g}, \qquad (20)$$

or, assuming $a$ as chosen, from (19):

$$\frac{n_1}{n_2} = a\frac{b + a\,g}{g - a\,b}. \qquad (21)$$

Substituting (1) in (12):

$$x\,n_2^2 = \frac{(g - a\,b)^2}{a\,(b^2 + g^2)\,(g\,[1 - a^2] - 2\,a\,b)}. \qquad (22)$$

$$x\,n_1\,n_2 = \frac{(g - a\,b)\,(b + a\,g)}{(b^2 + g^2)\,(g\,[1 - a^2] - 2\,a\,b)}. \qquad (23)$$

$$x\,n_1^2 = \frac{a\,(b + a\,g)^2}{(b^2 + g^2)\,(g\,[1 - a^2] - 2\,a\,b)} \qquad (24)$$

As instance, let:

1.

$$Y = 1 + 3\,j,$$

$$x\,n_2^2 = \frac{(1 - 3\,a)^2}{10\,a\,(1 - 6\,a - a^2)}.$$

$$x\,n_1\,n^2 = \frac{(1 - 3\,a)\,(3 + a)}{10\,(1 - 6\,a - a^2)}.$$

$$x\,n_1^2 = \frac{a\,(3 + a)^2}{10\,(1 - 6\,a - a^2)}.$$

2.

$$Y = 3 + 1\,j$$

$$x\,n_2^2 = \frac{(3 - a)^2}{10\,a\,(3 - 2\,a - 3\,a^2)}.$$

$$x\,n_1\,n_2 = \frac{(3 - a)\,(1 + 3\,a)}{10\,(3 - 2\,a - 3\,a^2)}.$$

$$x\,n_1^2 = \frac{a\,(1 + 3\,a)^2}{10\,(3 - 2\,a - 3\,a^2)}.$$

$x\,n_1^2$ and $x\,n_2$ must be positive. $x\,n_1\,n_2$ is positive if the coils on $M$ are wound in the same; negative, if in opposite direction.

It follows herefrom:

1.

$a > \sqrt{10} - 3$ $\qquad$ $x\,n_2^2 < 0$, $> .1623$ that is, impossible.

$\sqrt{10} - 3 > a > 0$: $\qquad$ $n_1\,n_2 > 0$: coil on $M$ wound in same direction.

$0 > a > -\sqrt{10}+3$: $x\,n_2^2 < 0$, that is, impossible.

$-(\sqrt{10}+3) > a$, $n_1 n_2 > 0$:

coils on $M$ in same direction.

This is the same range as above, but with $E_1$ and $E_2$ reversed.

2.

$a > \frac{1}{3}(\sqrt{10}-1)$, $x\,n_2^2 < 0$, $> .721$, that is, impossible.

$\frac{1}{3}(\sqrt{10}-1) > a > 0$, $n_1 n^2 > 0$,

coils on $M$ wound in same direction.

$0 > a > -\frac{1}{3}(\sqrt{10}+1)$, $x\,n_2^2 < 0$, that is, impossible.

$-\frac{1}{3}(\sqrt{10}+1) > a$, $n_1 n_2 > 0$,

coils on $M$ in same direction.

This is the same range as above, but with $E_1$ and $E_2$ reversed.

Instance:

| 1. | 2. |
|---|---|
| $a = .1$ | $a = .5$ |
| $x\,n_2^2 = 1.26$ | $x\,n_2^2 = 1.00$ |
| $x\,n_1 n_2 = .555$ | $x\,n_1 n_2 = .5$ |
| $x\,n_1^2 = .247$ | $x\,n_1^2 = .25$ |
| $n_2 = 2.25\,n_1$ | $n_2 = .2\,n_1$. |

Substituting in (14) (15):

$$\mathfrak{E}_1 = \frac{3.1 - .7j}{5.5 - 1.5j}e = .56\,e\,(1 + .044\,j).$$

$$\mathfrak{E}_1 = \frac{1.5\,(1-j)}{2.25 - 3.75j}\,e = .48.4\,(.97 + .24\,j)\,e.$$

$$\mathfrak{E}_2 = \frac{.07 + .31j}{5.5 - 1.5j}\,e = .056\,e\,(-.044 + j).$$

$$\mathfrak{E}_2 = \frac{.75\,(1+j)}{2.25 - 3.75j}e = .24.2\,(-.24 + .97\,j)\,e.$$

Substituting (22), (23) and (24) in (14) and (15) gives:

$$\mathfrak{E}_1 = \frac{e\,\{(b+ag)-j(g-ab)\}\,\{g[1-a^2]-2ab\}}{\{(1-a^2)[ag^2+bg(1-a^2)-ab^2]\}-j\{g^2[1+a^4]-2abg[1-a^2]+2a^2b^2\}} \quad (25)$$

$$\mathfrak{E}_2 = \frac{e\,a\,\{(g-ab)+j(b+ag)\}\,\{g[1-a^2]-2ab\}}{\text{same}}. \quad (26)$$

The current in circuit (1) is: $I_1 = E_1 Y$,
The current in circuit (2) is: $I_2 = E_2 Y$,
thus the total current:

$$I = I_1 + I_2 = (E_1 + E_2) Y,$$
$$= (E_1 + E_2)(g + j b). \qquad (27)$$

Substituting (25) and (26) in (27): (28)

$$Ie = \frac{\{2a(g^2-b^2)+j[g^2+b^2)(1-a^2)+4abg]\}\{g(1-a^2)-2ab\}}{\{1-a^2)(ag^2+by[1-a^2]-ab^2)\}-j\{g^2(1+a^4)-2abg(1-a^2)+2a^2b^2\}}.$$

As instance may be considered:

$$Y = g + j b = 3 + j. \qquad e = 100.$$

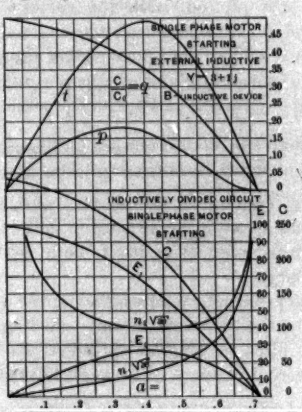


FIG. 16.

| $a$: | $x n_2^2 / \sqrt{x} n_2$ | $x n_1 n_2$ | $x n_1^2 / \sqrt{x} n_1$ | $E_1$ | $E_2$ | $I$ | $q = \frac{I}{I_0}$ | $p = \frac{E_1 E_2}{e^2}$ | $t = \frac{p}{q}$ |
|---|---|---|---|---|---|---|---|---|---|
| .721 | ∞ ∞ | ∞ | ∞ ∞ | 0 | 0 | 0 | 0 | 0 | 0 |
| .7 | 3.8 ∞ | 3.48 | 5.18 ∞ | 4.7 | 3.3 | 18.3 | .029 | .0016 | .055 |
| | 9.408 | | 2.276 | | | | | | |
| .6 | 1.335 | .933 | .655 | 27.7 | 16.6 | 102.5 | .162 | .046 | .285 |
| | 1.155 | | .809 | | | | | | |
| .5 | 1.00 | .50 | .25 | 48.5 | 24.2 | 171 | .270 | .118 | .438 |
| | 1.000 | | .500 | | | | | | |
| .4 | .98 | .333 | .113 | 66.0 | 26.4 | 226 | .356 | .174 | .490 |
| | .990 | | .336 | | | | | | |
| .3 | 1.14 | .242 | .051 | 78.0 | 24.2 | 264 | .406 | .180 | .445 |
| | 1.068 | | .226 | | | | | | |
| .2 | 1.38 | .181 | .021 | 89.0 | 17.8 | 287 | .453 | .155 | .342 |
| | 1.257 | | .145 | | | | | | |
| .1 | 3.03 | .136 | .006 | 95.5 | 9.6 | 303 | .478 | .091 | .190 |
| | 1.741 | | .077 | | | | | | |
| 0 | ∞ ∞ | 1.00 | 0 0 | 100 | 0 | 316 | .500 | 0 | 0 |

where:

$I_0 = 2\,e\sqrt{b^2 + g^2}$ = total current input of motor on quarter-phase circuit;

$\frac{E_1 E_2}{e^2}$ = torque, and

$t$ = apparent torque efficiency, in fraction of that of the same motor on quarter-phase circuit.

As seen before, with an inductive armature: $Y = 1 + 3j$, the conditions are much more unfavorable.

In Fig. 16 are plotted, with the values $a$ as abscissæ, the E. M. F.'s $E_1$ and $E_2$, the current $I$, the relative numbers of turns $n_1\sqrt{x}$ and $n_2\sqrt{x}$, and in the upper part of the Fig. 16 the ratio of currents $q$, of torques $p$, and of apparent torque efficiencies $t$, of this starting device, compared with the same motor on quarter-phase circuit.

## Internal Inductive Devices.

The exciting system of the motor consists of a stationary primary coil and a stationary secondary coil, short-circuited (or closed by an impedance) upon itself, both acting upon the revolving secondary.

The stationary secondary can either cover a part of the pole face excited by the primary circuit—shading coil,—or it can have the same pitch or angular spread as the primary, but be displaced therefrom in space by an angle less than 90°—accelerating coil.

### A.—Shading Coil.

Let, in Fig. 17,

$e$ = primary impressed E. M. F.

$I_0$ = " current.

$I_1$ = current in shading coil, reduced to primary number of turns.

$E_1$ = E. M. F. induced by flux through shading coil.

$E_0$ = " " " unshaded flux.

$Y_0 = \frac{1}{Z}$ = internal admittance of primary coil.[6]

6. Flux interlinked with primary coil only, but no other circuit and effective primary resistance.

$Y_1 = \frac{1}{Z_1}$ = internal admittance of shading coil.[1]

$Y$ = load admittance of armature.[2]

$Y_\infty$ = magnetizing admittance, total pole.[3]

$a$ = shaded portion of pole, as fraction of total pole,

thus: $1 - a$ = unshaded portion of pole, as fraction of total pole.

All quantities being reduced to primary number of turns.

It is, then:

Current in shading coil:

$$I_1 = E_1 \, Y_1 \tag{1}$$

For the unshaded section of pole, it is:

Current in primary coil:

$$I_0 = E_0 \left\{ (1 - a) \; Y + \frac{Y_\infty}{1 - a} \right\} \tag{2}$$

the first term representing the reaction of the induced armature currents, the last the exciting current of the pole arc $1 - a$.

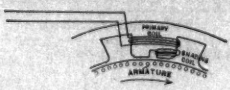


FIG. 17.

For the shaded section of pole, it is:

Current in primary coil:

$$I_0 = E_1 \left\{ a \; Y + \frac{Y_\infty}{a} + \; Y_1 \right\}. \tag{3}$$

the third term $I_1 = E_1 \; Y_1$ representing the reaction of the induced shading currents, the first and second term the same as in (2).

Combining (2) and (3), it is:

$$E_0 \left\{ (1 - a) \, Y + \frac{Y_\infty}{1 - a} \right\} - E_1 \left\{ a \; Y + \frac{Y_\infty}{a} + Y_1 \right\} = 0. \tag{4}$$

---

1. Flux interlinked with shading coil only, but no other circuit, and internal resistance, together with impedance of external circuit closing shading coil, if such exists.

2. $\frac{\text{Current in rev. secondary}}{\text{E. M. F. acting upon total pole}}$ with shading coil open circuited.

3. $\frac{\text{Current in primary of total pole}}{\text{E. M. F. at primary,}}$ with shading coil and armature open circuited.

It is, however, in primary circuit:

$$e = E_0 + E_1 + I_0 Z_0 = E_0 + E_1 + \frac{I_0}{Y_0}. \qquad (5)$$

Substituting (3) in (5):

$$e = E_0 + E_1 + E_1 \frac{Y_1 + a Y + \frac{Y_{00}}{a}}{Y_0},$$

or, transposed:

$$E_0 + E_1 \left\{ \frac{Y_0 + Y_1 + a Y + \frac{1}{a} Y_{00}}{Y_0} \right\} = e. \qquad (6)$$

(4) and (6) combined give:

$$E_0 = \frac{e Y_0 \left\{ a Y + Y_1 + \frac{1}{a} Y_{00} \right\}}{Y_0 (Y + Y_1) + (1-a)(a Y + Y_1) + \frac{Y_{00}}{a(1-a)} \left\{ Y_0 + a Y_1 + (1 - 2a + 2a^2) Y + Y_{00} \right\}} \qquad (7)$$

$$E_1 = e \frac{Y_0 \left\{ (1-a) Y + \frac{1}{1-a} Y_{00} \right\}}{\text{same denominator}}. \qquad (8)$$

Phase displacement angle $(E_1, E_0) = \omega$:

$$A (\cos \omega + j \sin \omega) = \frac{E_1}{E_0} = \frac{(1-a) Y + \frac{Y_{00}}{1-a}}{a Y + Y_1 + \frac{Y_{00}}{a}}. \qquad (9)$$

Assuming the shading coil to cover half the poleface, that is:

$$a = 1 - a = \tfrac{1}{2},$$

and neglecting the primary impedance:

$$Z_0 = \frac{1}{Y_0} = 0,$$

it is:

primary current: $$I_0 = \frac{E_0}{2} (Y + 4 Y_{00}), \qquad (10)$$

and:

$$\left. \begin{aligned} E_0 &= \frac{e}{2} \frac{Y + 2 Y_1 + 4 Y_{00}}{Y + Y_1 + 4 Y_{00}}, \\ E_1 &= \frac{e}{2} \frac{Y + 4 Y_{00}}{Y + Y_1 + 4 Y_{00}}. \end{aligned} \right\} \qquad (11)$$

Phase angle: $$A (\cos \omega + j \sin \omega) = \frac{Y + 4 Y_{00}}{Y + 2 Y_1 + 4 Y_{00}} \qquad (10)$$

Substituting: $Y + Y_1 + 4\,Y_\infty = Y^1$ (13)

it is:

$$I_0 = \frac{E_0}{2}(Y^1 - Y_1), \tag{14}$$

$$\left.\begin{aligned} E_0 &= \frac{e}{2}\frac{Y^1 + Y_1}{Y_1} = \frac{e}{2}\left(1 + \frac{Y_1}{Y^1}\right), \\ E_1 &= \frac{e}{2}\frac{Y^1 - Y_1}{Y^1} = \frac{e}{2}\left(1 - \frac{Y_1}{Y^1}\right), \end{aligned}\right\} \tag{15}$$

$$I_0 = \frac{e}{4}\frac{Y^{1\,2} - Y_1^2}{Y^1}. \tag{16}$$

$$A(\cos\omega + j\sin\omega) = \frac{Y^1 - Y_1}{Y^1 + Y_1}. \tag{17}$$

Substituting now $Y = g + j\,b$ etc., and eliminating imaginary quantities, gives:

$$\left.\begin{aligned} E_0 &= \frac{e}{2\,y^1}\sqrt{\left(g^1 + g_1\right)^2 + \left(b^1 + b_1\right)^2}, \\ E_1 &= \frac{e}{2\,y^1}\sqrt{\left(g^1 - g_1\right)^2 + \left(b^1 - b_1\right)^2}. \end{aligned}\right\} \tag{18}$$

$$I_0 = \frac{e}{4\,y^1}\sqrt{\left[\left(g^1 + g_1\right)^2 + \left(b^1 + b_1\right)^2\right]\left[\left(g^1 - g_1\right)^2 + \left(b^1 - b_1\right)^2\right]} \tag{19}$$

$$A(\cos\omega + j\sin\omega) = \frac{(g^1 - g_1) + j\,(b^1 - b_1)}{(g_1 + g_1) + j\,(b^1 + b_1)}, \tag{20}$$

hence:

$$\tan\omega = 2\,\frac{g_1\,b^1 - g^1\,b_1}{y^{1\,2} + y_1^2}. \tag{21}$$

Since the denominator in (21) can never $= 0$, $\omega$ can never $= 90°$. That is:

It is not possible to produce exact quadrature flux by means of this shading coil.

$\omega = 0$, for: $g_1\,b^1 - g^1\,b_1 = 0$, or:

$$\frac{g_1}{b_1} = \frac{g^1}{b^1}, \text{ or } \frac{g_1}{y_1} = \frac{g^1}{y^1}. \tag{22}$$

That is:

If the power factor of the shading coil: $\frac{g_1}{y_1}$, equals the power factor $\frac{g^1}{y^1}$ of the term: $Y + Y_1 + 4\,Y_\infty$, the shaded flux comes in phase with the unshaded flux, and the torque becomes $= 0$.

If: $\frac{g_1}{y_1} > \frac{g^1}{y^1}$, $\omega > 0$, that is, the torque of the motor is from the direction of the unshaded portion, towards the shaded portion of the pole, and the shading coil is a lagging coil.

If: $\frac{g_1}{y_1} > \frac{g^1}{y}$, $\omega > 0$, that is, the torque is from the shaded towards the unshaded portion of the motor pole, and the shading coil is an accelerating coil.

Hence, with a high resistance low reactance shading coil the motor armature turns towards the shading coil, as shown by the arrow in Fig 17, with a high reactance low resistance shading coil it turns in the opposite direction.

The torque of an induction motor is the sum of the vector products of primary counter E. M. F.'s times secondary currents induced by their quadrature E. M. F.'s, thus in the present instance.

$$T = /\mathrm{E}_0\, \mathrm{I}_1{}^0/ + /\mathrm{E}_1\, \mathrm{I}_0{}^0/, \tag{23}$$

where:

$\mathrm{I}_0{}^0$ = current induced in the armature section covered by unshaded,

$\mathrm{I}_1{}^0$ = current induced in the armature section covered by shaded portion of pole, reduced to primary number of turns.

It is however:

$$\left.\begin{aligned} \mathrm{I}_0{}^0 &= \mathrm{E}_0\, Y\,(1-a) \\ \mathrm{I}_1{}^0 &= \mathrm{E}_1\, Y\, a, \end{aligned}\right\} \tag{24}$$

or, at $a = \frac{1}{2}$:

$$\left.\begin{aligned} \mathrm{I}_0{}^0 &= \frac{\mathrm{E}_0\, Y}{2}, \\ \mathrm{I}_1{}^0 &= \frac{\mathrm{E}_1\, Y}{2}. \end{aligned}\right\} \tag{25}$$

Substituting:

$$\tan a = \frac{b}{g}, \tag{26}$$

where $a$ = lag angle of induced armature currents $I_0{}^0$ and $I_0{}^1$ behind their inducing E. M. F.'s $E_0$ and $E_1$,

since $\omega$ = phase displacement between $E_0$ and $E_1$,

the phase displacement between $E_0$ and $I_1{}^0$ is $\omega + a$,

" " " " $E_1$ and $I_0{}^0$ is $\omega - a$,

hence, substituting the real values in (23), with (25):

$$T = \frac{E_0 E_1 y}{2} \left\{ \sin(\omega + a) + \sin(\omega - a) \right\}$$
$$= E_0 E_1 y \cos a \sin \omega, \qquad (27)$$

since, from (26):

$$\cos a = \frac{g}{y},$$

it is:

$$T = E_0 E_1 g \sin \omega. \qquad (28)$$

the torque of the motor.

Substituting (18) and (21) in (28), it is:

$$T = \frac{e^2 g}{4 y'^2} \sqrt{[(g' + g_1)^2 + (b' + b_1)^2]\,[(g' - g_1)^2 + (b' - b_1)^2]} \times \frac{2 (g_1 b' - g' b_1)}{\sqrt{(y'^2 + y_1^2)^2 + 4 (g_1 b' - g' b_1)^2}}. \qquad (29)$$
$$= \frac{e^2 g (g_1 b' - g' b_1) R}{2 y'^2 \sqrt{(y'^2 + y_1^2)^2 + 4 (g_1 b' - g' b_1)^2}}.$$

Since:

$$R^2 = [(g' + g_1)^2 + (b' + b_1)^2]\,[g' - g_1)^2 + (b' - b_1)^2]$$
$$= (y'^2 + y_1^2)^2 - 2\,[(g' g_1 + b' b_1)^2 - (g' b_1 - g_1 b')^2],$$

it is, substituted in (29):

$$T = \frac{e^2 g (g_1 b' - g' b_1)}{2 y'^2} \times \sqrt{\frac{1 - \frac{2\,[(g' g_1 + b' b_1)^2 - (g' b_1 - g_1 b')^2]}{(y'^2 + y_1^2)^2}}{1 + \frac{4 (g_1 b' - g' b_1)^2}{(y'^2 + y_1^2)^2}}} \qquad (30)$$

Thus, approximately:

$$T = \frac{e^2 g (g_1 b' - g' b_1)}{2 y'^2}. \qquad (31)$$

The apparent input of the motor is:

$$Q = e I_0. \qquad (32)$$

Substituting (19), in (32), it is:

$$Q = \frac{e^2 R}{4 y'}. \qquad (33)$$

Thus, the apparent torque efficiency, by substituting (29):

$$t = \frac{T}{Q} = \frac{2 g (g_1 b' - g' b_1)}{y' \sqrt{(y'^2 + y_1^2)^2 + 4 (g_1 b' - g' b_1)^2}} \qquad (34)$$

or:

$$t = \frac{g_1}{y} \frac{1}{\sqrt{1 + \left(\frac{y^{1\,2} + y_1^2}{2\,(g_1 b^1 - g^1 b_1)}\right)^2}}. \qquad (35)$$

Neglecting the second term in the denominator of (34), it is:

$$t = \frac{2\,g\,(g_1 b^1 - g^1 b_1)}{y^1\,(y^{1\,2} + y_1^2)}. \qquad (36)$$

As seen, the apparent torque efficiency is very small under any circumstance, except when $b_1 < 0$, that is, the shading coil is closed by capacity.

As instances may be considered the following conditions:

APPARENT TORQUE EFFICIENCY.

| | Shading Coil: $Y_1$ | Secondary: $Y$ | Primary Excitation: $Y_{00}$ | Hence: Resultant: $Y^1$ | $t$ in Approx: (36) | $t$ %, exact: (35) | $\frac{t}{g}$ | |
|---|---|---|---|---|---|---|---|---|
| 0 | .5 | 1.0 | .05+.2$j$ | 1.7+ .8$j$ | +11.3 | +11.0 | 11.0 | |
| | | .8+ .6$j$ | ..... | 1.5+1.4$j$ | +12.2 | +11.7 | 14.6 | |
| | | .6+ .8$j$ | ..... | 1.3+1.6$j$ | +10.3 | + 9.7 | 16.2 | |
| | | .1+1.0$j$ | ..... | .8+1.8$j$ | + 2.2 | + 2.0 | 20.0 | |
| 37 | .4+.3$j$ | 1.0 | ..... | 1.6+1.1$j$ | − 1.03 | − 1.03 | ..... | |
| | | .8+ .6$j$ | ..... | 1.4+1.7$j$ | + 3.7 | + 3.7 | ..... | |
| | | .6+ .8$j$ | ..... | 1.2+1.9$j$ | + 4.0 | + 4.0 | ..... | |
| | | .1+1.0$j$ | ..... | .7+2.1$j$ | + 1.16 | + 1.15 | ..... | |
| 53 | .3+.4$j$ | 1.0 | ..... | 1.5+1.2$j$ | − 6.3 | − 6.3 | ..... | |
| | | .8+ .6$j$ | ..... | 1.3+1.8$j$ | + .28 | + .28 | ..... | |
| | | .6+ .8$j$ | ..... | 1.1+2.0$j$ | + 1.53 | + 1.53 | ..... | |
| | | .1+1.0$j$ | ..... | .6+2.9$j$ | + .74 | + .73 | ..... | |
| 90 | .5$j$ | 1.0 | ..... | 1.9+1.3$j$ | −20.0 | −18.9 | −18.9 | |
| | | .8+ .6$j$ | ..... | 1.0+1.9$j$ | − 7.6 | − 7.5 | − 9.4 | |
| | | .6+ .8$j$ | ..... | .8+2.1$j$ | − 4.0 | − 4.0 | − 6.7 | |
| | | .1+1.0$j$ | ..... | .3+2.3$j$ | − .15 | − .15 | − 1.5 | |
| −37 | .4−.3$j$ | 1.0 | ..... | 1.6+ .5$j$ | +26.5 | +24.3 | | Capacity in Shading Circuit. |
| | | .8+ .6$j$ | ..... | 1.4+1.1$j$ | +32.7 | +30.2 | | |
| | | .6+ .8$j$ | ..... | 1.2+1.3$j$ | +17.7 | +15.8 | | |
| −53 | .3−.4$j$ | 1.0 | ..... | 1.5+ .4$j$ | +34.8 | +30.6 | | |
| | | .8+ .6$j$ | ..... | 1.3+1.1$j$ | +27.2 | +23.8 | | |
| | | .6+ .8$j$ | ..... | 1.1+1.2$j$ | +20.3 | +18.0 | | |

These values are recorded as curves in Fig. 18. As seen, they are extremely small, and somewhat better, although still low, by the use of capacity in the shading circuit.

The torque is in some instances in the one, in others in the other direction, according to the relative power factors of admittances $Y_1$ and $Y^1$.

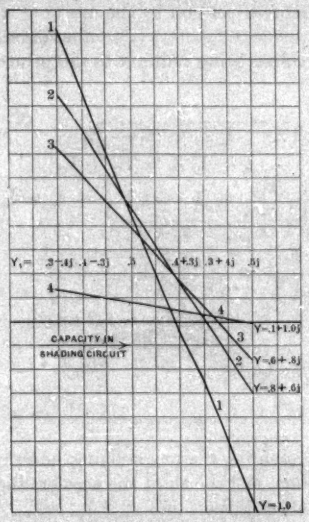


FIG. 18.—Single Phase Induction Motor. Starting Shading Coil.

Magnetizing Admittance of Motor: $Y = .05 + 2j$.

| | CURVE |
|---|---|
| Load Admittance of Armature: $Y = 1.0$ | (1) |
| $.8 + .6j$ | (2) |
| $.6 + .8j$ | (3) |
| $.1 + 1.0j$ | (4) |
| Shading Coil Admittance $Y_1 = .3 - .4j$ | |
| $.4 - .3j$ | |
| $.5$ | |
| $.4 + .3j$ | |
| $.3 + .4j$ | |
| $.5j$ | |

*B* Inductive Device.

## B. ACCELERATING COIL.

Let, in Fig. 19,

$e$ = primary impressed E. M. F.,

$I_0$ = primary current,

$I_1$ = current in accelerating coil,

$\frac{1}{Y_0} = Z_0$ = impedance of primary coil,[1]

$Z_1$ = " " accelerating coil,[1]

$Y_{00}$ = magnetic admittance per coil, primary or accelerat. coil,[1]

$Y$ = load admittance of armature,[1]

$a$ = ratio of widths of sections of primary coil ($2 \div 1$ in Fig. 19),

$E_1$ = E. M. F. induced per coil by the one $(1 - a)$,

$E_2$ = " " " " " " other ($a$) section of primary coil, all these values being reduced to the primary coil by the ratio of turns.

It is then:

Accelerating coil: $I_1 Z_1 = E_1 - E_2$ (1)

Primary coil: $e_0 - I_0 Z_0 = E_1 + E_2$ (2)

} E. M. F.

Section 1: $E_1 \left\{ \frac{Y_{00}}{1-a} + (1-a)\, Y \right\} + I_1 = I_0$ (3)

" 2: $E_2 \left\{ \frac{Y_{00}}{a} + a\, Y \right\} - I_1 = I_0$ (4)

} M. M. F.

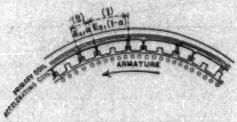


FIG. 19.

the first terms representing the magnetizing current, the second the armature reaction, and the third the accelerating coil.

(3) and (4) combined gives:

$$2\, I_0 = E_2 \left\{ \frac{Y_{00}}{a} + a\, Y \right\} + E_1 \left\{ \frac{Y_{00}}{1-a} + (1-a)\, Y \right\} \quad (5)$$

$$2\, I_1 = E_2 \left\{ \frac{Y_{00}}{a} + a\, Y \right\} - E_1 \left\{ \frac{Y_{00}}{1-a} + (1-a)\, Y \right\} \quad (6)$$

(5) and (6) substituted in (1) and (2), and transposed, gives after some transformations:

$$E_1 = 2\, e \frac{2 + Z_1 \left\{ \frac{Y_{00}}{a} + a\, Y \right\}}{\left\{ 2 + Z_0 \left[ \frac{Y_{00}}{1-a} + (1-a)\, Y \right] \right\} \left\{ 2 + Z_1 \left[ \frac{Y_{00}}{a} + a\, Y \right] \right\} + \left\{ 2 + Z_0 \left[ \frac{Y_{00}}{a} + a\, Y \right] \right\} \left\{ 2 + Z_1 \left[ \frac{Y_{00}}{1-a} + (1-a)\, Y \right] \right\}} \quad (7)$$

1. See footnotes 0 to 3 under "Shading Coil."

$$E_2 = 2\,e\frac{2 + Z_1\left\{\frac{Y_\infty}{1-a} + (1-a)\,Y\right\}}{\text{same denominator}}. \tag{8}$$

Assuming the accelerating coil of the same impedance as the primary coil:

$$Z_1 = Z_0 = Z,$$

and $a = \frac{1}{3}$, or 60° space displacement, it is:

$$E_1 = \frac{e}{2 + Z\left\{\frac{Y_\infty}{1-a} + (1-a)\,Y\right\}}, \tag{9}$$

$$E_2 = \frac{e}{2 + Z\left\{\frac{Y_\infty}{a} + a\,Y\right\}}, \tag{10}$$

and $a = \frac{1}{3}$:

$$E_1 = \frac{6\,e}{12 + Z\,[9\,Y_\infty + 4\,Y]}, \tag{11}$$

$$E_2 = \frac{3\,e}{6 + Z\,[9\,Y_\infty + Y]}. \tag{12}$$

Phase displacement between $E_1$ and $E_2$: $\angle(E_2, E_1) = \omega$:

$$A\,(\cos\omega + j\sin\omega) = \frac{12 + Z\,(9\,Y_\infty + 4\,Y)}{12 + 2\,Z\,(9_\infty + Y)}, \tag{13}$$

substituting for $Z_1\ Y_1\ Y_\infty$:

$$A\,(\cos\omega + j\sin\omega) = \frac{12+(r-jx)\,[(4g+9g_\infty)+j(4b+9b_\infty)]}{12+2\,(r-jx)\,[(g+9g_\infty)+j\,(b+9b_\infty)]},$$

$$= \frac{[12+r(4g+9g_\infty)+x(4b+9b_\infty)]+j[r(4b+9b_\infty)-x(4g+9g_\infty)]}{[12+2r(g+9g_\infty)+2x(b+9b_\infty)]+2j[r(b+9b_\infty)-x(g+9g_\infty)]} \tag{14}$$

Substituting:

$$\left.\begin{array}{ll} 4\,g + 9\,g_\infty = g_1, & 4\,b + 9\,b_\infty = b_1, \\ 2\,(g + 9\,g_\infty) = g_2, & 2\,(b + 9\,b_\infty) = b_2, \end{array}\right\} \tag{14}$$

it is:

$$A\,(\cos\omega + j\sin\omega) = \frac{(12 + r\,g_1 + x\,b_1) + j\,(r\,b_1 - x\,g_1)}{(12 + r\,g_2 + x\,b_2) + j\,(r\,b_2 - x\,g_2)}$$

$$= \frac{[(12+rg_1+xb_1)(12+rg_2+xb_2)+(rb_1-xg_1)(rb_2-xg_2)]+j\,[(rb_1-xg_1)(12+rg_2+xb_2)-(rb_2-xg_2)(12+rg_1+xb_1)]}{(12+rg_2+xb_2)^2+(rb_2-xg_2)^2},$$

hence:

$$\tan = \frac{(rb_1 - xg_1)(12 + rg_2 + xb_2) - (rb_2 - xg_2)(12 + rg_1 + xb_1}{(12 + rg_1 + b_1)(12 + rg_2 + xb_2) + (rb_1 - xg_1)(rb_2 - xg_2)}$$

$$= \frac{12r(b_1 - b_2) - 12x(g_1 - g_2) + (r^2 + x^2)(b_1 g_2 - g_1 b_2)}{144 + 12r(g_1 + g_2) + 12x(b_1 + b_2) + g_1 g_2 + b_1 b_2)(r^2 + x^2)}.$$

That is:

$$\tan \omega = \frac{12r(b_1 - b_2) - 12x(g_1 - g_2) + Z^2(b_1 g_2 - g_1 b_2)}{144 + 12r(g_1 + g_2) + 12x(b_1 + b_2) + Z^2(g_1 g_2 + b_1 b_2)}. \quad (15)$$

Since the denominator can never $= 0$, it is not possible to produce exact quadrature flux:

$$\omega = 90°.$$
$$\omega = 0$$

for:

$$12\,r\,(b_1 - b_2) - 12\,x\,(g_1 - g_2) + z^2\,(b_1\,g_2 - g_1\,b_2) = 0, \quad (16)$$

or approximately:

$$12\,r\,(b_1 - b_2) - 12\,x\,(g_1 - g_2) = 0,$$

$$\frac{x}{r} = \frac{b_1 - b_2}{g_1 - g_2},$$

substituting (14):

$$\frac{x}{r} = \frac{2\,b - 9\,b_\infty}{2\,g - 9\,g_\infty} = \frac{b + 4.5\,b_\infty}{g + 4.5\,g_\infty}. \quad (17)$$

That is, if the displacement angle of the accelerating coil:

$$\tan x_1 = \frac{x}{r},$$

equals the displacement angle of the armature plus 4.5 times the magnetizing current, the torque $= 0$. On one side of this point the torque is in the one, on the other side in the opposite direction.

This condition is essentially the same as found in "A, Shading Coil," thus the further discussion can be omitted.

### C.—Monocyclic Starting Devices.

The monocyclic starting devices consist in producing externally to the motor a system of polyphase E. M. F.'s with single-phase flow of energy, and impressing it upon the motor, which is wound as polyphase motor.

Such a polyphase system of E. M. F.'s with single-phase flow of energy has been called a monocyclic system. It essentially consists, or can be resolved into, a main or energy E. M. F., in phase

with the flow of energy, and a teaser or wattless E. M. F., in quadrature with the flow of energy and thus with the main E. M. F.

By combining these E. M. F.'s, compound systems of polyphase E. M. F.'s can be produced, as for instance a three-phase triangle with one side as teaser, and the other two sides as resultants of teaser and main (Fig. 20, the direction of the main E. M. F. being given by the arrow), or a three-phase triangle with one side as main, and the other two sides as resultants of main and teaser (Fig. 21), or a quarter-phase system of E. M. F.'s with the diagonal as main, and the sides as resultants of main and teaser (Fig. 22):

When derived from a monocyclic generator, the wattless feature of the teaser voltage is secured by its internal reactions, thus not as complete as when the teaser voltage is derived from the main voltage by phase displacing devices. Thus only the latter case will be discussed.

FIG. 20.

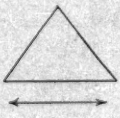

FIG. 21.

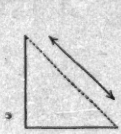

FIG. 22.

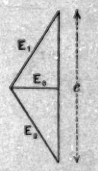


FIG. 23.

If across the single-phase mains of E. M. F. $e$, two impedances of different inductance factors are connected, the E. M. F.'s across these two impedances, $E_1$ and $E_2$, are displaced from each other, thus forming with the main E. M. F. $e$ on E. M. F. triangle, (Fig. 23.)

The altitude of this triangle, or the E. M. F. $E_0$ between the common connection of the two impedances and a point inside of the main E. M. F. $e$ (its middle, if the impedances are equal), is an E. M. F. in quadrature with $e$, and a teaser voltage, that is, when current is derived from this E. M. F., it droops.

Such an E. M. F. triangle thus constitutes a monocyclic system, or a polyphase system of E. M. F.'s with single-phase flow of power, and can be used for starting a suitable induction motor, as for instance a motor with main and teaser winding, or a motor with three-phase winding.

Let, in Fig. 24:

$e$ = E. M. F. between single-phase lines,
$I$ = current in single-phase lines,
$Y$ = admittance of motor between single-phase lines 1 and 2,
$I^1$ = current flowing through this admittance between 1 and 2,
$Y_1$, $E_1$, $I_1$ = admittance, E. M. F. and current in impedance 1,
$Y_2$, $E_2$, $I_2$ = admittance, E. M. F. and current in impedance 2,
$Y_0$, $E_0$, $I_0$ = admittance, E. M. F. and current of motor circuit in quadrature to the main motor circuit, or from 3 towards (1, 2),
$E_0 = e_0' + j e_0''$,
$I_1^1$ = current entering terminal 1 of motor,
$I_2^1$ = current leaving terminal 2 of motor,
$I_{s1}$ and $I_{s2}$ = current flowing from 3 to 1 and 2 respectively.

The directions in which these E. M. F.'s and currents are counted, as shown by the arrows in Fig. 24, the E. M. F. triangle shown in Fig. 25.

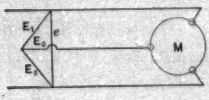


FIG. 25.

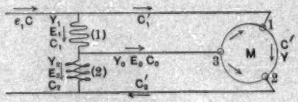


FIG. 24.

It is then:

$$\left.\begin{aligned} E_1 + E_2 &= e, \\ E_2 - E_1 &= 2\,E_0, \end{aligned}\right\} \qquad (1)$$

$$I_1 - I_2 = I_0, \qquad (2)$$

$$\left.\begin{aligned} I_1 &= Y_1 E_1, \\ I_2 &= Y_2 E_2, \end{aligned}\right\} \qquad (3)$$

$$I_0 = Y_0 E_0, \qquad (4)$$

Substituting (3) and (4) in (2):

$$Y_1 E_1 = Y_2 E_2 = Y_0 E_0, \qquad (5)$$

From (1) we get:

$$\left.\begin{aligned} E_1 &= \frac{e}{2} - E_0, \\ E_2 &= \frac{e}{2} + E_0, \end{aligned}\right\} \qquad (6)$$

Substituting (6) in (5), and transposing:

$$E_0 = \frac{e}{2}\frac{Y_1 - Y_2}{Y_1 + Y_2 + Y_0} \Bigg\} \quad (7)$$

Substituting (7) in (6):

$$\left.\begin{aligned} E_1 &= \frac{e}{2}\frac{2\,Y_2 + Y_0}{Y_1 + Y_2 + Y_0} \\ E_2 &= \frac{e}{2}\frac{2\,Y_1 + Y_0}{Y_1 + Y_2 + Y_0} \end{aligned}\right\} \quad (8)$$

Substituting (7) and (8) in (3) and (4):

$$\left.\begin{aligned} I_1 &= \frac{e}{2}\frac{Y_1(2\,Y_2 + Y_0)}{Y_1 + Y_2 + Y_0} \\ I_2 &= \frac{e}{2}\frac{Y_2(2\,Y_1 + Y_0)}{Y_1 + Y_2 + Y_0} \end{aligned}\right\} \quad (9)$$

$$I_0 = \frac{e}{2}\frac{Y_0(Y_1 - Y_2)}{Y_1 + Y_2 + Y_0} \Bigg\} \quad (10)$$

It is, from diagram:

$$I_1 = e\,Y, \quad (11)$$

$$\left.\begin{aligned} I_1^1 &= I^1 - \frac{I_0}{2} \\ I_2^1 &= I^1 + \frac{I_0}{2} \end{aligned}\right\} \quad (11)$$

$$I = I_1^1 + I_1 = I_2^1 + I_2. \quad (13)$$

Substituting in these equations (12, 13) the equations (9, 10, 11) gives:

$$\left.\begin{aligned} I_1^1 &= \frac{e}{4}\left\{4\,Y - \frac{Y_0(Y_1 - Y_2)}{Y_1 + Y_2 + Y_0}\right\} \\ I_2^1 &= \frac{e}{4}\left\{4\,Y + \frac{Y\,(Y_1 - Y_2)}{Y_1 + Y_2 + Y_0}\right\} \end{aligned}\right\} \quad (14)$$

$$I = \frac{e}{4}\left\{4\,Y + \frac{4\,Y_1\,Y_2 + Y_1\,Y_0 + Y_2\,Y_0}{Y_1 + Y_2 + Y_0}\right\} \quad (15)$$

The total volt-ampere input of motor and starting device is:

$$\begin{aligned} Q &= e\,I, \\ &= \frac{e^2}{4}\left[4\,Y = \frac{4\,Y_1\,Y_2 + Y_1\,Y_0 + Y_2\,Y_0}{Y_1 + Y_2 + Y_0}\right] \end{aligned} \quad (16)$$

that of the motor alone:

$$Q^1 = e\,I^1 + E_0\,I_0. \quad (17)$$

In a polyphase motor, the volt-ampere input of the two quadrature fluxes is equal, thus in the motor $M$ on polyphase circuit it is:

$$Q_0 = 2\,e\,I^1 = 2\,e^2\,y. \tag{18}$$

Thus the ratio of apparent inputs:

$$\mathrm{I} = \frac{Q}{Q_0} = \frac{1}{8}\,y\left[4\,Y + \frac{4\,Y_1\,Y_2 + Y_1\,Y_0 + Y_2\,Y_0}{Y_1 + Y_2 + Y_0}\right]. \tag{19}$$

The ratio of torque of this motor $M$ with monocyclic starting device to the torque of the same motor on polyphase circuits is obviously the ratio of the quadrature flux produced by the teaser voltage $E_0$ to the quadrature flux in the polyphase motor, that is the main flux produced by $e$.

Hence, if $q$ = ratio of number of effective turns of the cross circuit $Y_0$ to the main circuit $Y$, a polyphase flux would require in this cross circuit the impressed E. M. F. $q\,e$, and thus, if $e_0$ = imaginary or quadrature component of the cross E. M. F. $E_0 = e_0 + j\,e_0''$, the ratio of torque by monocyclic starting device, $T$, to polyphase torque $T_0$, is:

$$\frac{T}{T_0} = \frac{e_0''}{q\,e}. \tag{20}$$

Hence the relative apparent torque efficiency, or ratio of apparent torque efficiencies:

$$t = \frac{T}{T_0} \times \frac{Q_0}{Q}. \tag{21}$$

If the motor $M$ is wound with main circuit and teaser circuit of one quarter as many turns of the same size as those of the main circuit, $q = \frac{1}{4}$,

if it is wound with two equal circuits, as quarter-phase motor, it is: $q = 1$ or: $q\ \frac{1}{2}$,

if it is wound as three-phase motor, it is: $q = \frac{1}{2}\ \sqrt{3} = .866$.

These three are the most important arrangements.

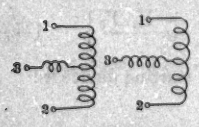


FIG. 26.

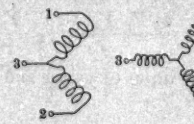


FIG. 27.

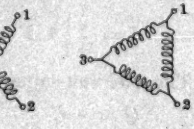


FIG. 28.

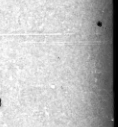


FIG. 29.




FIG. 30.

| | | | |
|---|---|---|---|
| FIG. 26. | Monocyclic Motor: | $q = \frac{1}{4}$. | $Y_0 = 4\,Y$, |
| FIG. 27. | Quarter-phase Motor: | $q = 1$. | $Y_0 = Y$ |
| FIG. 28. | " " | $q = \frac{1}{4}$. | $Y_0 = 4\,Y$, |
| FIGS. 26 to 30. | Three-phase Motor: | $q = \frac{1}{4}\ \sqrt{3}$. | $Y_0 = 4/3\ Y$. |

Substituting these values in equations (7) (10) (15), it is:

1. Monocyclic Motor: $q = .25, \quad Y_0 = 4\ Y.$

$$E_0 = \frac{e}{2} \frac{Y_1 - Y_2}{Y_1 + Y_2 + 4\ Y},$$

$$I_0 = 2\ e \frac{Y (Y_1 - Y_2)}{Y_1 + Y_2 + 4\ Y}. \tag{22}$$

$$I = e \frac{4\ Y^2 + 2\ Y\ Y_1 + 2\ Y\ Y_2 + Y_1\ Y_2}{Y_1 + Y_2 + 4\ Y}.$$

2. Quarter-phase Motor, connected for $q = 1, \ Y_0 = Y$:

$$E_0 = \frac{e}{2} \frac{Y_1 - Y_2}{Y_1\ Y_2 + Y},$$

$$I_1 = \frac{e}{2} \frac{Y (Y_1 - Y_2)}{Y_1 + Y_2\ Y}. \tag{23}$$

$$I = \frac{e}{4} \frac{4\ Y^2 + 5\ Y\ Y_1 + 5\ Y\ Y_2 + 4\ Y_1\ Y_2}{Y_1 + Y_2 + Y}.$$

3. Quarter-phase Motor, connected for $q = .5, \quad Y_0 = 4\ Y$:

$$E_0 = \frac{e}{2} \frac{Y_1 - Y_2}{Y_1 + Y_2 + 4\ Y},$$

$$I_0 = 2\ e \frac{Y (Y_1 - Y_2)}{Y_1 + Y_2 + 4\ Y}, \tag{24}$$

$$I = e \frac{Y^2 + 2\ Y\ Y_1 + 2\ Y\ Y_2 + Y\ Y_2}{Y_1 + Y_2 + 4\ Y}.$$

4. Three-phase Motor: $q = \frac{1}{2}\sqrt{3}, \quad Y_0 = 4/3\ Y.$

$$E_0 = \frac{e}{2} \frac{Y_1 - Y_2}{Y_1 + Y_2 + 4/3\ Y} = \frac{3\ e}{2} \frac{Y_1 - Y_2}{3\ Y_1 + Y_2 + 4\ Y},$$

$$I_0 = \frac{2\ e}{3} \frac{Y (Y_1 - Y_2)}{Y_1 + Y_2 + 4/3\ Y} = 2\ e \frac{Y (Y_1 - Y_2}{3\ Y_1 + 3\ Y_2 + 4\ Y} \tag{25}$$

$$I = \frac{e}{4} \left\{ 4\ Y + \frac{4\ Y_1\ Y_2 + 4/3\ Y\ Y_1 + 4/3\ Y\ Y_2}{2\ Y_1 + Y_2 + 4/3\ Y} = \right.$$

$$e \frac{4\ Y^2 + 4\ Y\ Y_1 + 4\ Y\ Y_2 + 3\ Y_1\ Y_2}{3\ Y_1 + 3\ Y_2 + 4\ Y}$$

The most important combinations of impedances are:
Resistance — inductance, as the simplest device.

Capacity — inductance, as giving the highest cross E. M. F. $E_0$ and thus best starting torque and torque efficiencies, and

Combinations of both, as:

Non inductive resistance — reactive coil,

Electrolytic condenser — reactive coil, etc.

The three-phase motor with monocyclic starting device, consisting of resistance — inductance, is the only type of single-phase induction motor which has found an extensive commercial application, in sizes from $\frac{1}{4}$ H. P. to 100 H. P., and thus will be more fully discussed.

### (*a*). *Resistance — Inductance.*

Let, in a three-phase $\triangle$ connected motor, $Y^1 = g^1 + j\,b^1$ = admittance per motor circuit. (In a $Y$ connected motor with admittance $Y^{11}$ per circuit, the $\triangle$ admittance, or admittance reduced to $\triangle$ connection, would be: $Y^1 = \frac{1}{3}\ Y^{11}$.)

Let $Y_1 = r, \quad Y_2 = +j\,a.$

Introducing the substitution:

$$Y^1 = a\,A = a\,(\gamma + j\,\beta), \qquad \vartheta = \sqrt{\gamma^2 + \beta^2},$$

where $\vartheta$ is the ratio of the absolute admittance per motor circuit to that of the resistance or inductance of the starting device.

In the three-phase motor with $\triangle$ admittance $Y^1$ it is:

$Y = 1.5\ Y^1 = 1.5\ a\,A$ = admittance between single-phase mains,

$Y_0 = 2\ Y^1 = 2\ a\,A$ = admittance of cross circuit, and

$q = \frac{1}{2}\sqrt{3}$ = ratio of effective turns of teaser or cross circuit to main circuit.

Substituting these values in the preceding equations, it is:

$$\mathrm{E}_0 = \frac{e}{2}\,\frac{1-j}{1+j+2\,A},$$

$$\mathrm{E}_1 = e\,\frac{j+A}{1+j+2\,A},$$

$$\mathrm{E}_2 = e\,\frac{1+A}{1+j+2\,A},$$

$$\mathrm{I}_1 = e\,a\,\frac{j+A}{1+j+2\,A},$$

$$\mathrm{I}_1 = j\,e\,a\,\frac{1+A}{1+j+2\,A},$$

$$\mathrm{I}_0 = e\,a\,A\,\frac{1-j}{1+j+2\,A},$$

$$I^1 = 1.5\, e\, a\, \Delta,$$

$$I_1^1 = e\, a\, \Delta\, \frac{1 + 2j + 3\Delta}{1 + j + 2\Delta},$$

$$I_2^1 = e\, a\, \Delta\, \frac{2 + j + 3\Delta}{1 + j + 2\Delta},$$

$$I = e\, a \frac{j + 2\Delta + 2j\Delta + 3\Delta^2}{1 + j + 2\Delta}.$$

$$avg\ I_1, I_2 = \frac{I_1 + I_2}{2} = \frac{e\, a}{2} \frac{2j + \Delta}{1 + j + 2\Delta}\,(1 + j).$$

Quadrature component of $E_0$:

$$e_0^1 = \frac{e}{2}\left[\frac{1 - j}{1 + j + 2\Delta}\right]^1.$$

Relative torque:

$$t = \frac{T}{T\triangle} = \frac{e_0^1}{q\ e} = \frac{1}{\sqrt{3}}\left[\frac{1 - j}{1 + j + 2\Delta}\right]^1,$$

where $T\triangle$ = three-phase torque.

In the following Table II. are calculated the relative torque,

TABLE II.

RESISTANCE — INDUCTANCE.

$Z = .1 - .3\,j.$ $\quad Y = .01 + .1\,j.$ $\quad e = 110.$

| $R$: | $T\triangle$: | $I\triangle$: | $PF\triangle$: | $IF\triangle$: | $ATE\triangle$: | $y^1$: | $Y^1$: | $e_0^1$: | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | $a=.5$: | $a=1$: | $a=2$: | $a=4$: |
| 0 | 8.80 | 176 | .315 | .948 | .153 | 1.60 | .305+1.515$j$ | 9.4 | 16.4 | 25.8 | 52.3 |
| .15 | 18.15 | 160 | .490 | .870 | .340 | 1.46 | .715+1.27 $j$ | 10 6 | 17.8 | 27 0 | 52.6 |
| .5 | 24.75 | 124 | .740 | .670 | .615 | 1.13 | .836+ .757$j$ | 13.1 | 21.2 | 30.6 | 53.5 |
| 1.0 | 21.90 | 85 | .870 | .495 | .785 | .773 | .670+ .380$j$ | .... | 26.6 | .... | .... |
| 1.5 | 16.90 | 64 | .890 | .455 | .815 | .580 | .516+ .263$j$ | 20.7 | 30.3 | 39.0 | 54.6 |

| | $t$ | | | | $T$ | | | |
|---|---|---|---|---|---|---|---|---|
| $R$: | $a=.5$: | $a=1$: | $a=2$: | $a=4$: | $a=.5$: | $a=1$: | $a=2$: | $a=4$: |
| 0 | .099 | .172 | .272 | .548 | .87 | 1.51 | 2.40 | 4.02 |
| .15 | .111 | .188 | .283 | .551 | 2.02 | 3.41 | 5.13 | 10.00 |
| .5 | .134 | .223 | .322 | .561 | 3.40 | 5.52 | 7.95 | 13.90 |
| 1.0 | | .273 | | | | 6.05 | | |
| 1.5 | .227 | .318 | 41 | .573 | 3.67 | 5.40 | 6.92 | 9.70 |

| | $I$ | | | | $I$ | | | |
|---|---|---|---|---|---|---|---|---|
| $R$: | $a$=.5: | $a$=1: | $a$=2: | $a$=4: | $a$=.5: | $a$=1: | $a$=2: | $a$=4: |
| 0 | 100+265$j$ | 121+283$j$ | 170+323$j$ | 314+445$j$ | 283 | 308 | 365 | 545 |
| .15 | 135+225$j$ | 156+244$j$ | 203+287$j$ | 343+415$j$ | 262 | 290 | 352 | 538 |
| .5 | 154+142$j$ | 175+163$j$ | 223+211$j$ | 353+343$j$ | 210 | 240 | 307 | 492 |
| 1.0 | | 149+105$j$ | | | | 182 | | |
| 1.5 | 103+63$j$ | 126+88$j$ | 176+140$j$ | 298+268$j$ | 121 | 154 | 229 | 401 |

| | $p$ | | | | $t/p$ | | | |
|---|---|---|---|---|---|---|---|---|
| $R$: | $a$=.5: | $a$=1: | $a$=2: | $a$=4: | $a$=.5: | $a$=1: | $a$=2: | $a$=4: |
| 0 | .537 | .538 | .692 | 1.03 | .184 | .203 | .394 | .532 |
| .15 | .545 | .606 | .732 | 1.12 | .203 | .310 | .386 | .493 |
| .5 | .565 | .646 | .827 | 1.32 | .343 | .345 | .390 | .425 |
| 1.0 | | .713 | | | | .390 | | |
| 1.5 | .930 | .803 | 1.17 | 2.09 | .344 | .397 | .350 | .273 |

in fraction of the polyphase motor torque $t$,

Torque, in synchronous K. W., $T$,

Total current input, $I$, inclusive starting device,

for the motor:

$$Z = .1 - .3j, \qquad Y = .01 + .1j, \qquad e = 110;$$

for the additional armature resistances:

$$R = 0, \quad .15, \quad .5, \quad 1.0, \quad 1.5,$$

and the constants of the starting device:

$$a = .5, \quad 1, \quad 2, \quad 4.$$

Since a single-phase motor is generally operated at 30 per cent. higher density than the same motor as polyphase motor, and is rated, at this higher density, at $\frac{3}{4}$ the output of the polyphase motor, in Table III. the corresponding values are given, and plotted in Figs. 31, 32 and 33 for this motor as three-phase motor, and as single-phase motor at 30 per cent. higher magnetic density, that is rewound with $\frac{1}{1.3}$ times as many turns of 1.3 times the cross section, or of constants:

$$Z = .059 - .177j, \qquad Y = .0169 + .169j, \qquad e = 110.$$

$$R = 0, \quad .2535, \quad .845, \quad 1.69, \quad 2.535.$$

$$a = .845, \quad 1.69, \quad 3.38, \quad 6.76.$$

TABLE III.

| | *P F* | | | | | *A T E* | | | | | *T E* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *R:* | *a*=.5: | *a*=1: | *a*=2: | *a*=4: | △: | *a*=.5: | *a*=1: | *a*=2: | *a*=4: | △: | *a*=.5: | *a*=1: | *a*=2: | *a*=4: | △: |
| 0 | 35.2 | 29.3 | 46.5 | 58.0 | 31.5 | 2.8 | 4.5 | 6 0 | 8.15 | 15.3 | 7 9 | 11.4 | 12.9 | 14.1 | 31.5 |
| .15 | 51.5 | 54.0 | 57.6 | 62.0 | 49.0 | 6.9 | 10.6 | 13.1 | 16.8 | 34.0 | 13 4 | 19.6 | 22.8 | 27.0 | 69.3 |
| .3 | 73.5 | 73 5 | 72.7 | 72.0 | 74.0 | 15.0 | 21.2 | 24.0 | 26.2 | 61.5 | 20.4 | 28.8 | 33 0 | 36 4 | 83.0 |
| 1.0 | | 84.0 | | | 87.0 | | 30.6 | | | 78.5 | | 37.3 | | | 90.4 |
| 1.5 | 85.0 | 82.0 | 78.3 | 74.5 | 89.0 | 28.0 | 32.4 | 28 5 | 22.3 | 15.8 | 33.0 | 39.6 | 36.4 | 29.9 | 91.6 |

| | *T* | | | | | *C* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *R:* | *a*= 5: | *a*=1: | *a*=2: | *a*=4: | △: | *a*=.5: | *a*=1: | *a*=2: | *a*=4: | △: |
| 0 | 1.47 | 2.55 | 4.05 | 8.14 | 8.80 | 478 | 520 | 617 | 920 | 528 |
| .15 | 3 40 | 5.75 | 8.7 | 16.9 | 18.15 | 441 | 400 | 593 | 910 | 480 |
| .3 | 5 74 | 9.33 | 13.4 | 23.5 | 24.75 | 355 | 405 | 518 | 830 | 372 |
| 1.0 | | 10.9 | | | 21.90 | | 307 | | | 255 |
| 1.5 | 6.20 | 9.1 | 11.7 | 16.4 | 16.90 | 204 | 260 | 380 | 676 | 192 |

| | The three-phase motor is rated at: | The single-phase motor is rated at: |
|---|---|---|
| Output, full load | 15.45 K. w. | 10.8 K. w. |
| Full load current, total | 184 amps. | 142 amps. |
| No load current, total | 32 " | 51 " |
| Maximum output | 21.0 K. w. | 15.4 K. w. |
| Full load torque | 16.5 " | 11.1 " |
| Maximum torque | 24.75 " | 16.7 " |
| Maximum torque with starting device | *a*=.5: | 6.4 " |
| | 1: | 10.3 " |
| | 2: | 13.8 " |
| | 3: | 23.5 " |
| Current with starting device, at full load torque | *a*=1.2: | 305 amps. |

From Figs. 10, 11 and 12 of my previous paper on the polyphase induction motor are taken the values of the first columns of Table II:

$T_d$ = three-phase torque, or three times torque per circuit, as given in Fig. 2 of previous paper,

$I_d$ = three-phase current per circuit, absolute, from previous paper,

$P F_d$ = three-phase power factor, from previous paper,

$I F_d$ = three-phase induction factor, calculated from power factor, $= \sqrt{1 - P F_d^2}$,

$A T E_d$ = three-phase apparent torque efficiency, from previous paper.

Therefore:

$y^1$ = absolute admittance per three-phase △ circuit $= \frac{I_d}{e}$,

$Y^1$ = vector admittance per three-phase △ circuit

$= y^1 (P F_d + j 1 F_d)$,

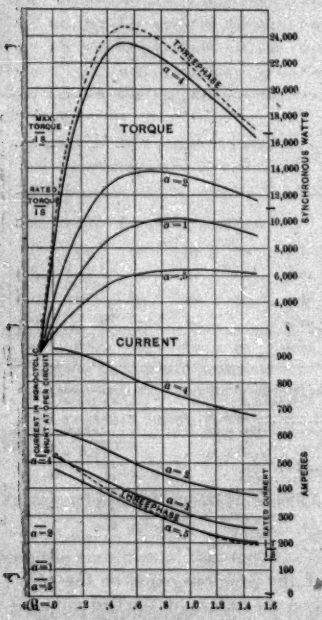


FIG. 31.—Single Phase Induction Motor.
Monocyclic Starting Device.
Resistance — Inductance
$Y_1 = a$. $Y_3 = j\,a$
Three Phase Motor
$Y = .01 + .1\,j$. $Z = .1 - .3\,j$. 110 Volts.

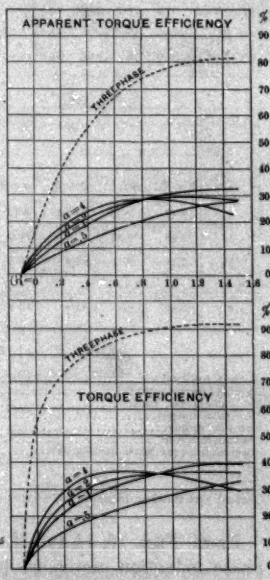


FIG. 32.—Single Phase Induction Motor.
Monocyclic Starting Device.
Resistance — Inductance
$Y = a$. $Y_3 = j\,a$.
Three Phase Motor.
$Y = .01 + .1\,j$. $Z = .1 - .3\,j$. 110 Volts

$e_0^1$ = CROSS E. M. F.

$$A = \frac{1}{\sqrt{3}}\left[\frac{1-j}{1+j+2A}\right]^1$$

$$T = t\,T_d,$$

$$I = e\,a\,A\,\frac{1-j}{1+j+2\,A},$$

$$I = /I/,$$

$p$ = ratio of single-phase and of three-phase current,

$\frac{t}{p}$ = ratio of single-phase and of three-phase apparent torque efficiency.

Herefrom, in Table III, for this motor as three-phase motor, and as single-phase motor at 30% higher magnetic density, and $e = 110$:

$P.\ F.$, = power factor, from $I$, for $a = .5, 1, 2, 4$, or rather: $a = .845, 1.69, 3.38$ and $6.76$, and for three-phase, $\triangle$:

$A.\ T.\ E. = \frac{t}{p}\ A.\ T.\ E._{\triangle}$, for $a = .5, 1, 2, 4$, or rather: $a = .845, 1.69, 3.38$ and $6.76$, and for three-phase, $\triangle$:

$T.\ E. = \frac{A.\ T.\ E.}{P.\ F.}$, for $a = .5, 1, 2, 4$, or rather: $a = .845, 1.69, 3.38$ and $6.76$, and for three-phase, $\triangle$:

Total current, $I$, for $a = .5, 1, 2, 4$, or rather: $a = .845, 1.69, 3.38$ and $6.76$, and for three-phase, $\triangle$: $= 3\ I_a$.

Torque $T$, Power input $P$, Volt-ampere input $Q$.

These values are plotted in Figs. 31, 32 and 33 with the external secondary resistance $R$ as abscissæ. The resistance $R$ refers to the three-phase motor, and thus in the single-phase motor is $\frac{1}{1.3^2}$ as high, due to the higher density.

As seen from these tables and diagrams, the torque and current curves, as function of the secondary resistance, have with the monocyclic starting device $b\,e\,j$ resistance—inductance the same general shape as with the polyphase motor.

Choosing in the single-phase motor a 30% higher density, the maximum available starting torque is about the same as with the three-phase motor (thus, when rating the single-phase motor at about $\frac{2}{3}$ the capacity of the same motor as polyphase motor, a larger percentage of full lead torque than on polyphase circuit.) This starting torque however requires a very much larger current than with the polyphase motor, from two to three times as much at $a = 4$.

The single-phase motor can start with about the same current as the polyphase motor, but gives in this case only about one third the torque, at $a = .5$.

In the diagram (Fig. 31) are marked, also the current consumed in the monocyclic starting device (at 45° lag) with the motor disconnected therefrom, and the full load currents of the motor

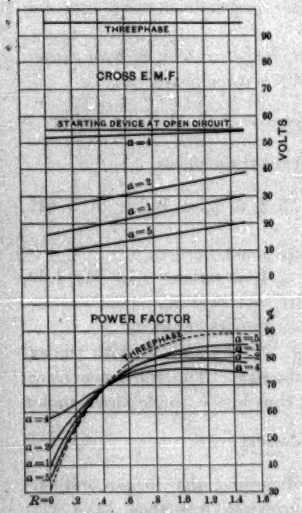


FIG. 33.—Single Phase Induction Motor.
Starting Device.
Resistance — Inductance
$Y_1 = a$. $Y_2 = j\,a$.
Three Phase Motor.
$Y = .01 + .1j$. $Z = .1 - 3j$. 110 Volts.

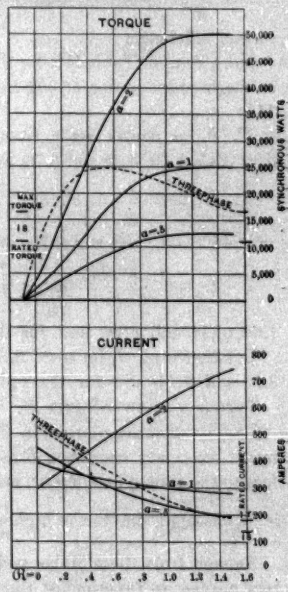


FIG. 34.—Single Phase Induction Motor.
Monocyclic Starting Device.
Condensance — Inductance
$Y_1 = j\,a$. $Y_2 = -j\,a$.
Three Phase Motors
$Y = .01 + .1j$. $Z = .1 - 3j$. 110 Volts.

without starting device, the motors being rated at $\frac{2}{3}$ their maximum torque as full load torque.

### b.—CAPACITY.—INDUCTANCE.

Let, in a three-phase motor, $Y^1 = g^1 + j\,b^1$ = admittance per motor circuit, reduced to $\triangle$ connection.

Let:

$$Y_1 = +j\,a,$$
$$Y_2 = -j\,a.$$

Substituting:

$Y^1 = a\,\Delta = a\,(\gamma + j\,\beta)$, $\vartheta = \sqrt{\gamma^2 + \beta^2}$, it is, in the three-phase motor:

$$Y = 1.5\ Y^1 = 1.5\ a\ \Delta,$$
$$Y_0 = 2\ Y^1 = 2\ a\ \Delta,$$
$$q = \tfrac{1}{2}\sqrt{3}.$$

Hence, substituting these values in the general equation, it is:

$$E_0 = \frac{e}{2}\frac{j}{\Delta},$$

$$E_1 = \frac{e}{2}\frac{\Delta - j}{\Delta},$$

$$E_2 = \frac{e}{2}\frac{\Delta + j}{\Delta},$$

$$I_1 = \frac{e\,a}{2}\frac{1 + j\,\Delta}{\Delta},$$

$$I_2 = \frac{e\,a}{2}\frac{1 - j\,\Delta}{\Delta},$$

$$I_0 = j\,e\,a,$$

$$a\,v\,g\,I_1, I_2 = \frac{I_1 + I_2}{2} = \frac{e\,a}{2}\cdot\frac{\cdot}{\Delta},$$

$$e_0 j = \frac{e}{2}\left[\frac{j}{\Delta}\right]^1 = \frac{e\,\gamma}{2\,\vartheta^2},$$

$$I^1 = 1.5\,e\,a\,\Delta,$$

$$I_1^1 = \frac{e\,a}{2}\,(3\,\Delta - j),$$

$$I_2^1 = \frac{e\,a}{2}\,(3\,\Delta + j),$$

$$I = \frac{e\,a}{2}\frac{1 + 3\,\Delta^2}{\Delta}.$$

Relative torque:

$$t = \frac{T}{T_0} = \frac{e_0^1}{q\ e} = \frac{\gamma}{\vartheta^2\sqrt{3}}.$$

Relative current or volt-ampere input:

$$p\,\frac{e\,I}{3\,e\,I_d} = \frac{\frac{e^2\,a}{2\,\vartheta}\,[1+3\,\Delta^2]}{3\,e^2\,a\,\vartheta} = \frac{1}{6\,\vartheta^2}\,[1+3\,\Delta^2]$$

$$= \frac{1}{6}\,\sqrt{1+\frac{1}{\vartheta^4}+2\,\frac{\gamma^2-\beta^2}{\vartheta^4}}.$$

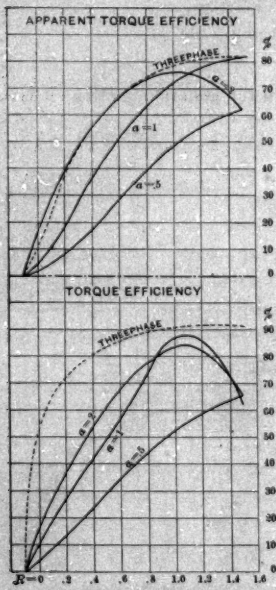


FIG. 35.—Single Phase Induction Motor.
Monocyclic Starting Device.
Condensance — Inductance
$Y_1 = j\,a.$ $Y_2 = -j\,a.$
Three Phase Motor
$Y = .01 + .1\,j.$ $Z = .1 - .3\,j.$ 110 Volts.

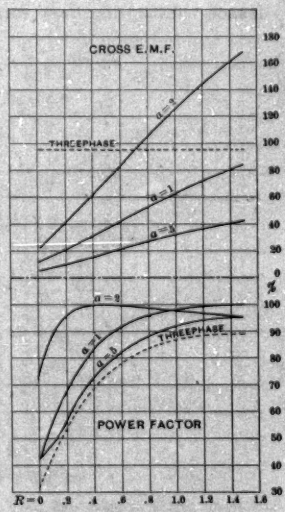


FIG. 36.—Single Phase Induction Motor.
Starting Device.
Condensance — Inductance
$Y_1 = j\,a.$ $Y_2 = -j\,a.$
Three Phase Motor
$Y = .01 + .1\,j.$ $Z = .1 - 3\,j.$ 110 Volts.

In Table IV. are calculated, and plotted on Figs. 34, 35 and 36, the values of:

Relative Torque: $t$.

Vector Current: $I$.

Relative Current: $p$.

Relative apparent torque efficiency: $\frac{t}{p}$

Power Factor.

Apparent torque efficiency.

TABLE IV.

CAPACITY — INDUCTANCE.

$Z = .1 - .3\,j.$ $\quad Y = .01 + .1\,j.$ $\quad e = 110.$

| | $t$ | | | $I$ | | |
|---|---|---|---|---|---|---|
| $R$: | $a$=.5: | $a$=1: | $a$=2: | $a$=.5: | $a$=1: | $a$=2: |
| 0 | .057 | .125 | .928 | $111+242\,j$ | $94+217\,j$ | $126+120\,j$ |
| .25 | .097 | .194 | .388 | $123+205\,j$ | $137+171\,j$ | $192+76\,j$ |
| .5 | .189 | .378 | .760 | $147+117\,j$ | $174+92\,j$ | $282-6.6\,j$ |
| 1.0 | .330 | .660 | 1.315 | $126+54\,j$ | $173+27\,j$ | $362-77\,j$ |
| 1.5 | .441 | .882 | 1.760 | $106+33\,j$ | $169+.6\,j$ | $423-128\,j$ |

| | $p$ | | | $t/p$ | | |
|---|---|---|---|---|---|---|
| $R$: | $a$=.5: | $a$=1: | $a$=2: | $a$=.5: | $a$=1: | $a$=2: |
| 0 | .500 | .445 | .33 | .113 | .282 | .690 |
| .25 | .500 | .457 | .435 | .194 | .425 | .893 |
| .5 | .506 | .530 | .76 | .365 | .716 | 1.00 |
| 1.0 | .538 | .688 | 1.45 | .614 | .962 | .910 |
| 1.5 | .580 | .880 | 2.31 | .762 | 1.00 | .765 |

| | $PF$ | | | | $ATE$ | | | | $TE$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $R$: | $a$=.5: | $a$=1: | $a$=2: | $\Delta$: | $a$=.5: | $a$=1: | $a$=2: | $\Delta$: | $a$=.5: | $a$=1: | $a$=2: | $\Delta$: |
| 0 | 41.8 | 40.0 | 72.4 | 31.5 | 1.72 | 4.3 | 10.4 | 15.3 | 4.1 | 10.5 | 14.5 | 51.5 |
| .25 | 51.4 | 62.5 | 92.3 | 49.0 | 6.6 | 14.4 | 30.3 | 34.0 | 12.8 | 23.1 | 32.8 | 69.5 |
| .5 | 78.0 | 88.4 | 100 | 74.0 | 22.4 | 44.0 | 61.5 | 61.5 | 28.7 | 49.8 | 61.5 | 83.0 |
| 1.0 | 92.0 | 98.6 | 97.8 | 87.0 | 48.2 | 75.5 | 71.4 | 78.5 | 52.5 | 76.7 | 73.0 | 90.4 |
| 1.5 | 95.5 | 100 | 95.7 | 89.0 | 62.0 | 81.5 | 69.3 | 81.5 | 65.0 | 62 | 65.2 | 91.6 |

| | *T* | | | | *I* | | | |
|---|---|---|---|---|---|---|---|---|
| | at 30 per cent. higher density: | | | | at 30 per cent. higher density: | | | |
| *R:* | $a=.5$: | $a=1$: | $a=2$: | $\Delta$: | $a=.5$: | $a=1$: | $a=2$: | $\Delta$: |
| 0 | .85 | 1.9 | 3.4 | 8.8 | 450 | 397 | 293 | 528 |
| .25 | 3.0 | 5.6 | 11.9 | 18.25 | 403 | 370 | 351 | 480 |
| .5 | 7.8 | 15.8 | 31.8 | 24.75 | 317 | 333 | 476 | 372 |
| 1.0 | 12.1 | 24.3 | 48.6 | 21.90 | 232 | 296 | 625 | 255 |
| 1.5 | 12.6 | 25.2 | 50.2 | 16.90 | 188 | 285 | 745 | 192 |

Torque efficiency of the three-phase motor:

$$Z = .1 - 3j,\ Y = .01 + 1j,\ e = 110,$$

with single-phase, capacity-inductance monocyclic starting devise of admittance: $a = .5$, $a = 1$, $a = 2$, and the values of torque.

Total current input of the motor as three-phase motor, and as single-phase motor at 30 per cent. higher density (that is, of the constants:

$$Z = .059 - .177j,\ Y = .0169 + .169j,\ e = 110,$$
$$a = .845,\ 1.69,\ 3.38.$$

As seen herefrom, by means of condensance-inductance as monocyclic starting device, the motor can be started with a torque far in excess to its maximum torque as three-phase motor; its power factor is higher than on three-phase circuit, its torque efficiency lower, and its apparent torque efficiency about equal to that of the three-phase motor.

While, however, with the resistance-inductance starting device, the general shape of the curves, as the change of torque, current, etc., with the secondary resistance of the motor, is of the same character as with the three-phase, showing a gradual approach from the single-phase shape towards that of the three-phase motor, with increasing $a$, it is in the capacity-inductance device essentially different, the more, the larger the capacity admittance $a$ is. For instance, with $a = 2$, the current is a minimum with short-circuited secondary, and rises very greatly with increasing secondary resistance—just the reverse of the three-phase motor. In general, by the use of capacity, the high range of the curves is short, that is, the curves are steeper.

A more complete discussion of the effect of capacity must, however be reserved for a separate paper.

c). *Polarization-Inductance.*—As further instance of the monocyclic starting device of single-phase motors are shown in Fig. 37, the curves of the three-phase motor,

$$Z = .1 - .3j,\ Y = .01 + .1j,\ e = 110,$$

or, as single-phase motor rewound for 1.30 times the magnetic density, as:

$$Z = .059 - .177j,\ Y = .0169 + .169j,\ e = 110,$$

with an inductance of .10 power factor and an electrolytic condenser, or series of polarization cells (which, as known, act like a

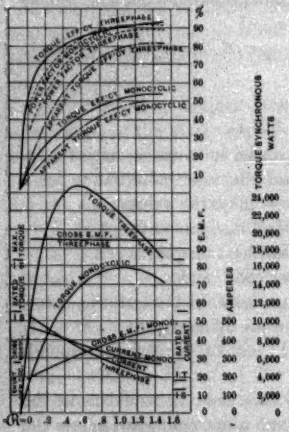


FIG. 37.—Single Phase Induction Motor.
Monocyclic Starting Device.
Electrolytic Condenser and Inductance.
$Y_1 = .17 + 1.52j$. $Y_2 = 1.18 - 1.18j$.
Three Phase Motor
$Y = .01 + .1j$. $Z = .1 - .3j$.

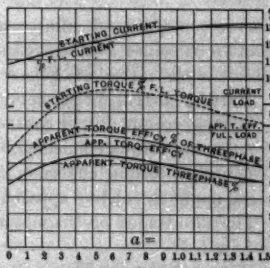
STARTING CURRENT
FULL LOAD CURRENT
STARTING TORQUE
FULL LOAD TORQUE
APPARENT TORQUE EFFICY. OF THREEPHASE
APP. TORQUE EFFICY.
APPARENT TORQUE THREEPHASE
a =
0 1 2 3 4 5 6 7 8 9 1.0 1.1 1.2 1.3 1.4 1.5

FIG. 38.—Single Phase Induction Motor. Starting.
Condenser of $-.375j$ Admittance in Shunt of Second. Self Induct of $+ aj$ Admittance in Shunt of Third Phase. Condenser Adjusted for 100 per cent. P. F. at ¾ Load. Total Admittance per △ Circuit: $Y_0 = .49 + .95j$.
Three-Phase Motor
$Y = .01 + .1j$. $Z = .1 - .3j$.

leaky condenser, or condenser of high power factor) of .707 power factor (45° lead), corresponding to $a = 1$ in the previous instances.

It is thus:

$$Y_1 = .1 + .9j,\ Y_2 = .707 - .707j,$$

or, reduced to the motor of 30 per cent. higher density.

$$Y_1 = .17 + .152j,\ Y_2 = 1.18 - 1.18j.$$

As seen, the curves show the same characteristics as the three-phase, and the resistance-inductance curves, being intermediate between them.

That is, torque, torque efficiencies, etc. are higher than with the resistance-inductance device, but lower than on three-phase circuit, except the power factor, which is higher.

These curves do not show any of the peculiarities of the condensance-inductance curves. That is, due to the high power factor of the electrolytic condenser, all resonance phenomena have disappeared.

In the preceding instances the absolute values of admittances $Y_1$ and $Y_2$ have been assumed as equal.

To investigate the effect of a change of one of these values, in Fig. 38 are plotted for:

$$Y_1 = +j\,a,$$
$$Y_2 = -.375j,$$

that is, constant condensance, and varied inductance, with the inductive reactance $a$ as abscissæ, and the total admittance per $\Delta$ circuit of the motor, of $Y^1 = .49 + .25j$, the values:

Starting current, in per cent. of full load current;

Starting torque, in per cent. of full load torque;

Apparent torque efficiency, in per cent., and

Apparent torque efficiency, in per cent. of that of the three-phase motor or relative apparent torque efficiency.

The admittance of the motor, $Y^1 = .49 + .25j$, corresponds to $\frac{3}{4}$ load.

The condensance, $Y_2 = -.375\,a$, is such as would make the power factor at $\frac{3}{4}$ load 100 per cent., when connected across the main circuit.

As seen, the changes with varying $a$ are comparatively small over a wide range.

The starting torque is a maximum at: $a = .65$.

The apparent torque efficiency is a maximum at: $a = .55$.

The relative apparent torque efficiency is a maximum at: $a = .5$.

The starting current increases

from 1.08 times three-phase current, at $a = 0$,

to 1.32 times three-phase current, at $a = 1.5$.

### §3. ACCELERATION WITH STARTING DEVICE.

The torque of the single-phase induction motor (without starting device) is proportional to the product of main flux or magnetic

flux produced by the primary impressed E. M. F. and the speed. Thus it is the same as in the polyphase motor at or very near synchronism, but falls off with decreasing speed and becomes zero at standstill.

To produce a starting torque, a device has to be used to impress an auxiliary magnetic flux upon the motor, in quadrature with the main flux in time and in space, and the starting torque is proportional to this auxiliary or quadrature flux. During acceleration or at intermediate speeds, the torque of the motor is the resultant of the main torque or torque produced by the primary main flux, and the auxiliary quadrature or starting flux. In general this resultant torque is not the sum of main and auxiliary torque, but less, due to the interaction between the motor and the starting device.

All the starting devices depend more or less upon the total admittance of the motor and its power factor. With increasing speed, however, the total admittance of the motor decreases and its power factor increases, and an auxiliary torque device suited for the admittance of the motor at standstill will not be suited any more for the changed admittance at speed.

The currents induced in the secondary by the main or primary magnetic flux are carried by the rotation of the motor more or less in quadrature position and thus produce the quadrature flux, giving the main torque as discussed in the first paragraph.

This quadrature component of the main flux induces an E. M. F. in the auxiliary circuit of the starting device and thus changes the distribution of currents and E. M. F.'s. in the starting device. The circuits of the starting device then contain besides the motor admittance and external admittance an active counter E. M. F., changing with the speed. Inversely the currents produced by the counter E. M. F. of the motor in the auxiliary circuits react upon the counter E. M. F., that is upon the quadrature component of main flux and change it.

Thus during acceleration we have to consider:

1st. The effect of the change of total motor admittance, and its power factor, upon the starting device.

2nd. The effect of the counter E. M. F. of the motor upon the starting device, and the effect of the starting device upon the counter E. M. F. of the motor.

1st. The total motor admittance and its power factor change very much during acceleration in motors with short-circuited low

resistance secondary. In such motors the admittance at rest is very large and its power factor low, and with increasing speed the admittance decreases and its power factor increases greatly. In motors with short-circuited high resistance secondary, the admittance also decreases greatly during acceleration, but its power factor changes less, being already high at standstill. Thus the starting device will be affected less. Such motors, however, are inefficient at speed. In motors with variable secondary resistance the admittance and its power factor are maintained constant during acceleration by decreasing the resistance of the secondary circuit in correspondence with the increasing counter E. M. F. Hence in such motors the starting device is not thrown out of adjustment by the changing admittance during acceleration, and they are thus preferable.

The investigation of the phenomena taking place during acceleration can be carried out in a similar manner as the investigation of the starting in paragraph 2, by considering, however, in the circuits besides their respective admittance or impedance the counter E. M. F.'s of the motor, or E. M. F.'s induced by the quadrature component of main flux. It would extend the paper too far, however, and thus must be postponed for a later occasion, and only some general conclusions drawn.

While any desired torque can be produced by the use of resistance and inductance and mutual induction as starting devices, the torque is necessarily far below that which can be produced in a polyphase motor with the same expenditure of volt-amperes or of watts, that is the apparent torque efficiency and the torque efficiency are lower. A much better starting torque efficiency, and apparent torque efficiency, under circumstances almost as good or better as in a polyphase motor, can be produced by the use of capacity. Capacity, however, is generally only suitable for a sine wave of E. M. F. and most generators do not give a sine wave, and in general the use of a wave differing from sine shape is preferable for other reasons, while for the motors proper, the wave shape is immaterial. Furthermore, looking over the curves of starting torque and apparent torque efficiency given in paragraph 2, the general feature is noticeable that capacity gives very high values for a very limited range only, but low values outside thereof. Thus, when securing high starting torque by means of capacity, with the change of admittance during acceleration, the conditions become unfavorable much faster than by the use of

resistance, inductance and mutual induction. Hence, while good starting torque can be produced by capacity, in general this torque is not maintained during acceleration up to the speed where the main torque of the motor is sufficient for further acceleration, and thus such a motor will start, but not run up to speed.

A. PHASE SPLITTING DEVICES.

In these the starting torque depends upon the relative proportion of the internal or motor admittance and the external admittance of the starting device, and is thus essentially affected by the change of admittance during acceleration in a motor with short-circuited low resistance secondary.

Thus, for instance, in Fig. 12, in a motor with impedance $Z = r - j\,x = 1 - 3j$ of each of the two primary coils at standstill, and inductive reactance $-j\,a$ in circuit of one of them, the apparent torque efficiency is very low and fairly constant, about 10 per cent. of that of a polyphase motor, for a wide range of reactance. During acceleration $r$ increases, $x$ decreases and the relative apparent torque efficiency approaches that of the motor $Z = 3 - 1j$, that is increases considerably. If, however, a great starting torque was produced by capacity as shown on the left side of Fig. 12, the relative apparent torque efficiency decreases during acceleration by passing from curve $Z = 1 - 3\,j$ towards curve $Z = 3 - 1\,j$.

The right hand side of Fig. 12 represents also the effect of resistance as a starting device, but with the two curves exchanged, the higher one representing $Z = 1 - 3\,j$, the lower one $Z = 3 - 1\,j$. Thus by using resistance $a$ in one of the motor circuits, a good starting torque, of nearly 50 per cent. of that of the polyphase motor of same volt-ampere input, is produced in a motor with low resistance secondary, but during acceleration it falls off due to the increase of power factor of the motor impedance, and the motor, while starting under good torque, will probably not be able to run up to speed, even under fairly light load.

Still more instructive is Fig. 13, two reactances, of which at least one is a capacity reactance.

As long as one of the reactances $-j\,b$ is inductive, the two curves; $Z = 1 - 3\,j$ representing a motor with low resistance armature, and $Z = 3 - 1\,j$ representing a motor with high resistance armature, and thus approximately the conditions of a motor with low resistance armature at speed, give fairly the same results, using, however, two capacity reactances, an enormous apparent

torque efficiency can be produced in starting, more than three times that of the polyphase motor, for $b = -2$. But with increasing power factor of impedance, that is during acceleration, this torque decreases to zero and then reverses, and $Z = 3 - 1j$ gives—61% of the polyphase apparent torque efficiency. Thus with this device, a low resistance motor will start with very powerful torque, but with increasing speed the torque produced by the starting device falls to zero and then reverses, and if the main torque of the motor is not very large at fairly low speeds, the motor will not run up to speed even at light load. Hence this device of two capacities in the two motor circuits is unsuitable for low resistance motors, although it gives a very powerful starting torque.

### B. INDUCTIVE DEVICES.

The inductive devices depend still more than the phase splitting devices upon the power factor of the motor admittance. As seen in paragraph 2, even the direction of the torque given by the starting device depends upon the power factor of the motor impedance.

For instance, the shading coil or the accelerating coil acts as retarding coil if its power factor is higher than a certain value depending upon that of the motor, etc., and as accelerating coil if its power factor is lower than this value. Hence with a very low reactance shading coil and a high reactance motor secondary, a fairly good torque is secured in starting, but this torque rapidly disappears and then reverses during acceleration, and this device can thus be used only with motors giving a main torque curve, however, requires fairly high armature resistance and thus gives low torque for the starting device. Hence, the shading coil is suitable only for very small motors starting under light load. The accelerating coil requires a high resistance secondary and when used with a motor with variable armature resistance, the motor can be made to start in one direction with the resistance in, but in the opposite direction by cutting out the resistance. In the latter case the motor obviously does not run up to speed, due to the reversal of the torque, except if after starting, the accelerating coil is cut out and the resistance cut in again. Torque curves of such a starting device will be shown in the following:

The monocyclic starting device, especially with resistance inductance, is scarcely at all affected by a change of the power fac-

tor of the admittance, and is improved by the decrease of admittance during acceleration, since a decrease of the motor admittance is equivalent to an increase of the admittance $A$, that is an increase of the teaser E. M. F. Thus it is the most satisfactory starting device, and is more fully discussed in the following.

2nd.—Effect of the counter E. M. F. of the motor upon the starting device and inversely.

In the phase splitting device, where a definite phase displacement is produced in circuits of different impedances, the induction of the counter E. M. F. in these circuits necessarily throws these circuits more or less out of phase. The same applies to external inductive devices.

Where several circuits are in multiple between the same primary mains, but displaced in position on the motor primary, the counter E. M. F's induced by the main flux of the motor in these circuits are displaced in phase from each other, and thus form more or less a short circuit, through the parallel coils, and thereby produce a current which reduces the counter E. M. F., and thus the main torque of the motor.

Still more is this the case with the internal inductive devices. The shading coil as well as the accelerating coil form a dead short circuit for the component of main flux in their direction, and thus practically annihilate this flux by the demagnetizing effect of the induced current. The result is a great decrease or even reversal of torque, and a falling off of output. For instance, in a single phase motor giving a maximum torque at 96 per cent. of synchronism of 217 lbs. at 1 ft. radius, with 93 amperes input at 220 volts, the maximum torque is reduced to 132 lbs. at 89 per cent. of synchronism, while the current is increased to 320 amperes, that is, the apparent torque efficiency has fallen off to 17.7 per cent. of its previous value, due to the short circuit caused by the inductive starting device of the accelerating coil. A number of torque curves of a motor with accelerating coil are shown in Fig 39 with different values of resistance in the secondary. This motor is the 1-8-30-900-110 of which the three-phase curves are shown in Fig. 1 and 2 of my previous paper on the polyphase induction motor, the single-phase curves in Fig. 9 and 10 and the curves with monocyclic starting device in Fig. 42 of this paper. The motor has a three-phase winding, of which one coil is excited as primary coil, one short-circuited and one open, thus the accelerating coil is displaced 60° from the main coil.

As seen, with short-circuited secondary, the starting torque is negative, 4 pounds and remains so up to a speed where the main torque of the motor overpowers the reverse torque of the starting device, or the latter disappears, due to the increase of the power factor of the motor, at 30 per cent. of synchronism. With medium resistance in the secondary the torque is 24 lbs. in starting, increases due to the increase of power factor of the motor as discussed before, reaches a maximum of 40 lbs., but then decreases

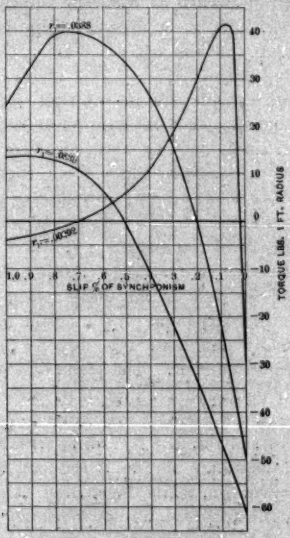


FIG. 39.
1.8 30-900-110. Single-Phase Motor.
Speed Curves with Inductive
Starting Device.
One Circuit Excited. One Circuit Short
Circuited. One Circuit Open.

again and reverses at 80 per cent. of synchronism, due to the short circuit. It becomes — 49 lbs. at synchronism. With high resistance in the secondary, the torque is a maximum of 14 lbs. at 1 ft. radius in starting, and then decreases due to the short-circuit, becomes zero at 48 per cent. of synchronism, and gives at synchronism —61 lbs. reverse torque.

These curves show best the interaction of the three effects; the main torque of the motor, zero at standstill and increasing with speed, the torque of the starting device, increasing in positive, or decreasing in negative direction, due to the change of power factor of the motor admittance, and the retarding effect of the short-circuit of main flux by the accelerating coil. The current in either of the curves is very large, thus the apparent torque efficiency low.

With the monocyclic starting device the effect of the counter E. M. F. is to raise the altitude of the monocyclic or teaser triangle,

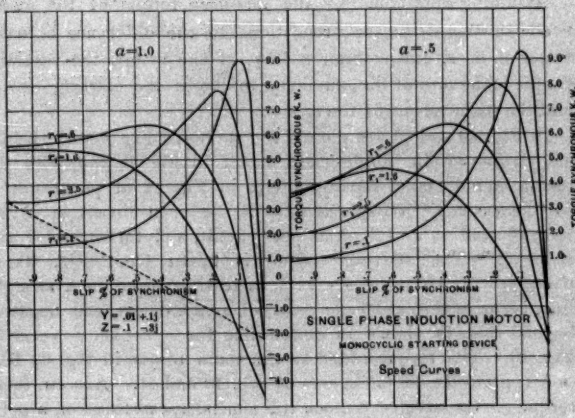


FIG. 40.




FIG. 41.

and thereby to increase the torque, up to the speed where the counter E. M. F. equals the altitude of the teaser triangle at open circuit. At this speed the teaser current becomes zero, (or very small since some current flows due to the difference of phase between teaser and counter E. M. F.), and the torque equals the main torque of the motor, that is the starting device has become ineffective. Beyond this the counter E. M. F. is higher than the impressed E. M. F. and current returns over the teaser, with the effect of reducing the torque of the motor by a partial short-circuit. With a motor of a three-phase winding with

two terminals connected to the single phase mains of E. M. F. $e$ and the third terminal to the teaser or intermediary connection of a resistance inductance monocyclic starting device, the maximum counter E. M. F. of the motor in the teaser circuit, neglecting internal drop, is $\frac{1}{2}\sqrt{3}\,e$ or .867 $e$. The maximum teaser voltage (at open circuit) is $e/2$, thus at $\frac{.5}{.867} = 58\%$ of synchronism the teaser current should cease and the monocyclic starting device become inactive and should be cut out. In reality, due to the drop of voltage in the motor, the point where the starting device becomes inactive lies at higher speed, from 60 to 65 per cent. of synchronism.

Assuming approximately the auxiliary torque of the monocyclic starting device as proportional to the difference between teaser E. M. F. and counter E. M. F. of the motor, the resultant torque is the sum of the auxiliary torque of the starting device and the main torque of the motor.

In Fig. 40 and 41 are plotted the curves of Fig. 40, 41, resultant torque of the single-phase motor in Fig. 6, 7, 8, with the resistance-inductance starting device in Fig. 31 of paragraph 2, for the secondary resistances $r_1 = .1$, or open circuit, $r_1 = .25$, $r_1 = .6$ and $r_1 = 1.6$ ohms per circuit, and for $a = 1.0$ and $a = .5$. The starting torque is taken from Fig. 31 in paragraph 2. It decreases with increasing speed in a straight line, becomes zero at 60 per cent. of synchronism, and negative beyond this, as shown in dotted line for $r_1 = .25$, $a = 1.0$ in Fig. 40. This torque added to the main torque in Fig. 6, Section 1, gives the torque curves of Fig. 40 and 41. As seen, these torque curves have very much the same shape as those of a polyphase motor, and change in the same manner with the change of secondary resistance, but do not become zero at synchronism, but at a definite speed below synchronism. The range of the curve near synchronism is of less interest, since the starting device is expected to be cut out of circuit between half-speed and two-third speed. It is of interest to compare these speed curves of the single-phase induction motor with monocyclic starting device in Figs. 40 and 41 with the speed curve of the same motor as polyphase motor in Fig. 10 and 11 of my previous paper, and as single-phase motor in Fig. 6, 7, 8, paragraph 1. As seen, with increasing $a$, the curves gradually change from those of the single-phase motor to those of the polyphase motor.

In Fig. 42 are shown the speed torque curves of a induction motor with monocyclic starting device as found by test, for three values of secondary resistance; short-circuit, medium and high resistance, together with the curves of the teaser current $I_0$ and the average of the two main currents $\frac{I_1^1 + I_2^1}{2}$, for high resistance in the secondary. As seen, the teaser current $I$ Fig. 42

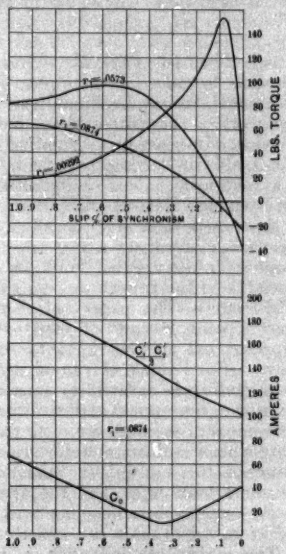


FIG. 42.
1.8-30-900-110. Speed Curves with Monocyclic Starting Device.
Resistance — Inductance
$Y_1 = 9.13.$ $Y_2 = .085 + 2.19\,f.$

decreases proportional to the speed, reaches a minimum at 65 per cent. of synchronism, and then increases again as return current, up to synchronism, exactly as to be expected theoretically.

Of the same motor the single-phase speed curves as shown in Figs. 9 and 10, Section 1, and the three-phase speed curves in Figs. 1 and 2 of my previous paper. All these three sets of curves are

taken at the same impressed voltage and frequency and are thus directly comparable. These tests, however, were made some years ago and thus the motor no longer represents the present state of the art.

Herefrom it appears, that of all single-phase induction motor starting devices the monocyclic device most nearly reproduces in starting and accelerating the conditions of the polyphase motor.

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Discussion.

The President:—This paper of Mr. Steinmetz is another valuable contribution to our Transactions, of which he has already given us several, and as he himself said, it is a continuation of previous work. The single-phase induction motor is rather a new device. It resembles somewhat the synchronous motor in being incapable of starting itself, and has small torque at speeds below synchronism. This limitation is brought out by the fact that Mr. Steinmetz devotes more than three-fourths of his paper to the starting devices and the effect that they have upon the action of the motor.

The paper is open for discussion. Will Dr. Kennelly favor us with a few remarks on the paper?

Dr. A. E. Kennelly:—The paper before us is certainly an abtruse one from the point of view of electric motors, because we generally suppose that an electric motor is a very simple piece of apparatus. But I think if you eliminate that major portion of the paper which is devoted to starting, the remnant of the paper, devoted to the normal action of the motor, resolves itself into one of comparative simplicity. As the President has remarked, the single-phase induction motor is a sort of intermediary device between the multiphase induction motor, with which we have some familiarity, and the synchronous single-phase machine. The single-phase machine must be brought up to speed in order to start, and must run strictly in synchronism or it will stop. The multiphase induction motor will not only start with a full load torque, but even with a torque considerably in excess of full load, and it will not run at exact synchronism. The intermediary device of the single-phase induction motor is one which will not start itself like its friend the single-phase synchronous motor, but is more easily started from rest, and when once started will run nearly in synchronism—more nearly in synchronism than the multiphase induction motor; but if pulled out of that synchronism by an overload, it has a greater tendency to stop than the multiphase induction motor. I think that the relation between the two machines is perhaps more readily comprehended by the mental device of assuming a double rotation of the field than in any other way. Perhaps I may

make that clearer by a diagram on the blackboard. If we consider the multiphase induction motor, we can assume that the field travels around at a perfectly definite rate, say right handedly. In the case of a single-phase induction motor, we can imagine that the alternating field consists of a double rotation so that the full field divides itself into two halves, one half going around right handedly and the other half going around left handedly, and each of these is a simple rotating field. Either of these components might be considered as a simple multiphase component in the absence of the other. You must have both components when you employ the single-phase current. The armature does not know which component to follow at starting, and you must determine which component it shall follow by giving it the initial impulse. Once it has been brought up nearly into synchronism with either component, it will follow that component as though the other did not exist, but when following the right hand component say at nearly synchronous speed, the left hand component will be rotating relatively to the armature at twice the speed. Therefore, if you superpose upon the ordinary multiphase induction motor a component of magnetic field revolving relatively to the armature, at double the speed, in the opposite direction, I think the results are those that are deduced by Mr. Steinmetz in his paper. The importance of course of supplying the necessary starting power to the motor renders it necessary to use these various devices which are described in the long appendix of the paper. These become of great importance in view of the small torque which the machine normally possesses at starting.

Mr. Steinmetz: I held the same opinion some time ago but in attempting to get results agreeing with experience from this theory of two magnetic fields of half intensity revolving in opposite directions, I have found that the theory does not represent the facts, and had to be given up, for several reasons.

1st—At standstill the magnetic field of the single-phase induction motor is undoubtedly alternating and can be resolved in two equal and oppositely revolving fields. At speed however, and especially at synchronism, one component has disappeared altogether and the other component is of full intensity, that is the field is a uniformly revolving field as shown by the fact that in a turn at right angles to the primary coil the same E. M. F. is induced, as in a turn parallel to the primary coil, but the E. M. F. is displaced in phase by 90°.

2nd—The current consumed by the single-phase induction motor when running light, contradicts the theory of the two oppositely revolving fields of half intensity. According to this theory the current running light should be equal to the sum of the exciting current of a polyphase motor of half impressed E. M. F. plus the current taken by a polyphase motor of half impressed E. M. F. when driven backward at full speed. This latter current,

however, is frequently many times larger than the current actually observed in a single-phase induction motor at synchronism.

3rd—The torque curve of the polyphase induction motor with low resistance secondary is as shown in $t$ in Fig. 1. The torque

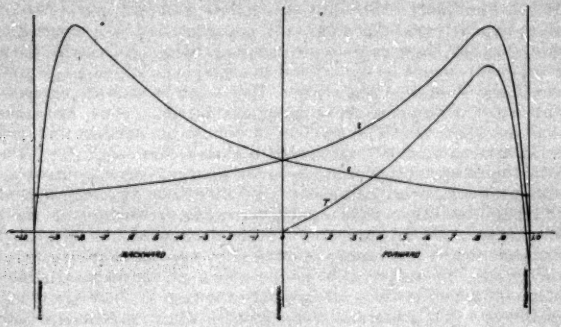


FIG. 1

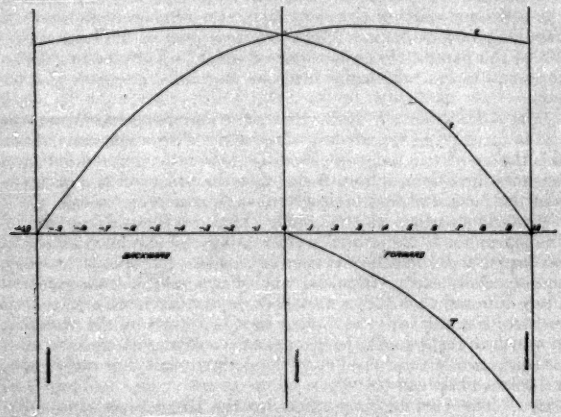


FIG. 2.

curve of the same motor with oppositely revolving field is as shown in Fig. 1 by $t^1$, and thus the torque curve of the single-phase induction motor should be the difference between $t$ and $t^1$, or $T$ in Fig. 1. It is in reality only approximately of similar

shape, in a motor with low resistance armature, but it entirely disagrees in a motor with high resistance armature. With very high resistance in the armature or secondary, the torque curve of the polyphase motor is as shown by $t$ in Fig. 2, with the maximum beyond standstill. The same motor with oppositely revolving field gives a torque curve $t^1$, and a single-phase motor should thus have as torque curve the difference $T = t - t^1$ as shown in Fig. 2, that is, should have negative torque over almost the entire range. This does not agree with experience, since we know that no matter how high the secondary resistance of the motor is, the torque still remains positive.

For these reasons, the theory of two equal and oppositely revolving fields must be given up, although it is still occasionally used in publications.

DR. KENNELLY:—The explanation which Mr. Steinmetz has given as to the discrepency between the facts based upon the double revolving theory, and the facts based upon a simple calculation of what takes place without referring to that theory, do not however include the consequences of the magnetizing influence of the currents in the revolving armature. In other words, he admits that when the armature is at rest, or when the machine is not in operation, as it were, the double revolving theory undoubtedly applies. Now, when you start the armature in operation you superpose upon that double revolution, which he admits is there, the effect of the currents in that revolving member and those effects, when superposed upon both halves and upon both members of the revolving coil, will, I believe, come to the same thing as the effects which he traces out directly in his paper. Of course you cannot neglect the effects of the currents in the revolving member upon both components, and take them into consideration, I think the results are the same as when the matter is treated in the able manner Mr. Steinmetz has treated it in his paper. It seems to me that the double revolving theory can be made to give the same results and is a simpler way of looking at the matter in theory, a simpler way of regarding the matter in mind. For example, all the main results mentioned by Mr. Steinmetz in his paper follow directly from the consequences, the natural consequences, of a double revolving field, the two components being in opposite directions, one right handed and the other left handed; namely, the power of the motor being so much reduced because the two components are divided, the relatively feeble torque of the motor, the double frequency component of current—all these three things are consistent with the double rotation theory, and while it is true as pointed out by Mr. Steinmetz that the double rotation theory simply considered is not apparently in accordance with that fact, still I take the position that if allowances are made for the effect of the revolving armature, if the armature reaction be taken into account, so to speak, that the double field theory can be

made to agree with the facts. The vector method is the only one up to the present date, that can simply and directly deal with ordinary current phenomena. Any one who will compare a single expression of alternating current motor phenomena as developed by the formula of vectors and developed by any other formula will be struck by the vast contrast presented by the two—one, simple, direct, compact; the other complex, vague and enormously extended.

MR. STEINMETZ:—I agree with Dr. Kennelly that by amending the theory to magneto-motive forces instead of magnetic fluxes, and taking in view the effect of induced currents, the theory of two opposite and equal rotations can be made to better agree with experience.

It would read then, that the primary induced circuit of the single-phase induction motor can be resolved into two equal and oppositely rotating M. M. F.'s, and the secondary circuit produces a M. M. F. proportional to the speed and additive to the one, subtractive to the other component of impressed M. M. F., thus annihilating the latter and doubling the former at synchronism.

PROF. W. S. FRANKLIN:—I am much pleased with the paper which I have heard discussed and partly read this evening, and I feel very thankful to Mr. Steinmetz for what he has given us. I wish to express my belief with Dr. Kennelly that the theory of the induction motor might easily be developed from the double rotation field theory. In regard to the vector method for studying alternating currents, a method which from the narrow point of view of a teacher I have always thought of as a graphical method, I may say that I have had considerable experience with it and I have always been skeptical of Mr. Steinmetz's method until I came to look over his book and some of his recent papers and I must say, that I am entirely converted to the author's point of view. I think the graphical method is inadequate except for the use we teachers make of it. I don't know whether Dr. Kennelly means Mr. Steinmetz's method when he speaks of the vector method, or whether he means simply the graphical method.

DR. KENNELLY:—I mean Mr. Steinmetz's method, which is a vector method and a most applicable one.

PROF. FRANKLIN:—I thought perhaps that Dr. Kennelly might include both the graphical and the symbolic method in the vector method and I don't know but it is quite proper to do so.

MR. STEINMETZ:—I want to draw attention to the investigation on page 148. If you will look at the diagram you will see there are nine different currents and still more E. M. F.'s. Now I defy anyone to get results by any other method of investigation in such a complex system without neglecting hysteresis and neglecting exciting current and neglecting self-induction and some other things, and ultimately get results which are entirely worthless. I may add that the method used here is used suc-

cessfully to predetermine the starting torque of induction motors and agrees with experience.

MR. TOWNSEND WOLCOTT:—If anybody will start with differential equations, even in the simplest case of alternating currents, he will find that he gets into deep water very quick. The only objection raised to this method is that it assumes sinusoidal variation. Now Mr. Steinmetz and others who have tried it in practice, say that the practical results justify the use of the method. Of course it is not theoretically correct, but if you get practical results from it, it is like a great many other things, justified in the application. Even in designing an ordinary continuous current dynamo you make assumptions about the flux in the field magnet which are not true, but at the same time you get practical results which are near enough true for all practical purposes. The true distribution of magnetic flux in the field magnet of an ordinary dynamo is unknown; that is, the analytical difficulties are so great that you cannot express it in a manner which is theoretically correct, but you can get at it near enough for practice with simple arithmetic. And it is just so with this method of Mr. Steinmetz. It is practically correct, whatever theoretical objection there may be to taking sine waves to represent a wave of any form.

PROF. FRANKLIN:—Mr. Wolcott has mentioned a thing which has often been in mind in contrasting the graphical method, or in its algebraic form, Mr. Steinmetz's method, with the analytical method or the use of differential equations in the study of the alternating current. We find that the method of differential equations is the one that does not assume a sinusoidal current, but, if you consider the matter, having a given saw-tooth form of E. M. F. your saw-tooth problem reduces itself to a series of sinusoidal problems. Now the problem to solve for the first member in that series is repeated for the second member and so on, and the problem which is based on any one of the harmonics of the alternating E. M. F. is precisely the problem which Mr. Steinmetz handles in his symbolic method, and which is handled by the ordinary method of graphical vectors, and we lose sight of the fact I think that the vector method is by using a series of diagrams applicable to any sort of a curve.

MR. WOLCOTT:—In Mr. Steinmetz's method, we assume the equivalent sine wave, which procedure has been objected to by some electricians, especially in England. The justification of it is in the practice as I understand it There may be mathematical objections to doing it in this case, but we also do it elsewhere. For instance when we introduce $2\pi$ into the expression for impedance we have already assumed the sinusoidal wave. That is if it happens to be some other shape, we assume an equivalent wave of simple sinusoidal form, instead of working out a whole series of terms, although it may not be correct from an analytical point of view.

DR. KENNELLY:—I hope to see the vector method used practically by engineers because of its simplicity, directness and usefulness. I think it is a great mistake to suppose that the vector geometrical method is limited to the conception and hypothesis of sinusoidal waves of electro-motive force, magnetism or current. When you go into an electric light station and see an alternating current dynamo that is producing a thousand volts by the voltmeter, and is producing say 30 amperes by the ammeter and 25 kilowatts by the wattmeter, you may form two suppositions; one is that you have an unknown, peculiar complex type of wave, and analyze it all out by supposing that you have a certain sine fundamental wave, and then the harmonics, and go through the various orders; or you can assume, which is equally nearly correct for all practical purposes, that the machine produces a sinusoidal E. M. F., a pure sine wave of a thousand volts, and that the load is such as to produce a lag of the current represented by the power factor determined from the wattmeter. The wattmeter shows that there is a power factor in the whole load of so much, and that will represent to a sinusoidal electromotive force a certain combination of inductance and resistance. It is, strictly speaking a false hypothesis, but it meets all the purposes of the case. The vector method may be considered to assume that you can treat the E. M. F. as a simple alternating sinusoidal electromotive force. By that means you eliminate all the complication incident to Fourier's method and you arrive at results which are practical and useful at once.

MR. STEINMETZ:—I fully agree that it is a matter of definition whether it is permissible to use equivalent sine waves or not. No matter what the shape of the wave is, we can always resolve it into a series of sine waves of different frequencies, and the theory of the equivalent sine wave merely means that the total energy is included in the fundamental wave and thereby all the higher harmonics made wattless waves. In consequence thereof in dealing with the fundamental wave only, as equivalent sine wave, we find results which check with experience, since this wave includes, and takes account of the total effect.

There are, however, undoubtedly cases where the theory of the equivalent sine wave cannot be used, as for instance in the case of the alternating arc, in which we have distortion between current and E. M. F. without phase displacement, and thus get a power factor without an angle of lag, or when attempting to calculate the angle of lag its sign becomes ambiguous.

Regarding, however, the use of Fourier's theorem or the use of equivalent sine waves, I have never seen any publication where real alternator waves, that is waves as given by machines in commercial operation, have been used, and the calculation carried out by the infinite series of Fourier and any results derived, so that it appears to me merely an academic discussion whether it is permissible to use equivalent sine waves or not, since you cannot use the complete series of Fourier anyway.

PROF. FRANKLIN:—If I may be permitted to make one more remark I wish to make clear what I said a moment ago, and that is that the vector method, including Mr. Steinmetz's method, is precisely the analytical method. It is the solution corresponding to the fundamental harmonic. You could get another corresponding to the second harmonic, etc. Indeed Mr. Steinmetz might write a Fourier series of papers, one for the first harmonic, one for the second and so on *ad infinitum* upon each subject which he brings before the INSTITUTE. I wish particularly to make the point that the analytical method with its Fourier's series is precisely the same thing as the vector method including Mr. Steinmetz's method, except that the latter as ordinarily carried out is not complete. Further, in regard to the conception of the equivalent wave. The use of this conception arises from the practical necessity of our being satisfied in our calculations with the results of one solution instead of a Fourier's series of solutions; and while in some respects the actual fundamental harmonic of the given E. M. F. or current may represent the actual state of affairs more closely than any other sinusoidal E. M. F. or currents, in other respects we know it does not, and we are justified in basing our single solution upon a sinusoidal E. M. F. or current which represents the state of affairs most closely in respect of those things which are of importance in the result.

MR. ELIAS E. RIES:—It seems to me that we have had a great deal of mathematics and quite a little theory here this evening and so far as it goes it has been very interesting. I note from Mr. Steinmetz's paper, however, that the single phase induction motor requires, in order to make it practical, a self-starting device or rather a separate starting device, and he goes into considerable detail as to various methods that might be used. Now, it is a well known fact that the single phase synchronous motor, which is one of the first of the successful alternating current motors of which we have any record, is deemed objectionable from a practical point of view, mainly because of the necessity of employing some means of starting it. I should like to inquire, since I do not see in the paper any reference made as to the particular type of motor, if an actual motor was used in making these tests, and I should like to ask what the practical results were that Mr. Steinmetz obtained in making these tests and whether or not he has come to the conclusion that the single phase induction motor, as a practical and available machine, is in advance of the single phase synchronous motor. Of course we understand that the induction motor possesses the advantage in most cases of dispensing with the commutator and other complicated mechanical devices, but as a practical machine, taking everything into consideration, has he found and is he in a position to state to the members of the INSTITUTE as a result of his experiments that this single phase induction motor, as compared

with the polyphase induction motor, is in a position where it can be applied to practical, every-day use. I think the INSTITUTE would be interested in hearing something on that side of the question.

MR. STEINMETZ:—I may state that the single phase induction motor is a standard article of manufacture and has been built and sold in sizes up to 100 H. P.

On high frequency circuits these motors are used only in sizes up to 15 H. P., not because large sizes cannot be built, but due to the nature of most high frequency circuits, in which parallel operation is not practiced, and usually a number of small generators are employed and circuits of limited capacity which no not permit the massing of larger loads as large single phase induction motors.

Low frequency single phase motors are used to a limited extent only, since most low frequency circuits or more modern installations employ either a polyphase system or a monoyclic system, and in either case the self-starting polyphase motor can be used.

The essential difference between the single phase induction motor and the single phase synchronous motor is that the single phase synchronous motor has to be brought up to complete synchronism and is thrown out of step, that is comes to a standstill if the frequency changes suddenly. Thus it cannot be used satisfactorily in these high frequency stations in which the alternators are not operated in parallel but with the changes of load the circuits are switched over from one machine to another machine, that is in most high frequency stations. In such stations, with increase of load on one generator, circuits operated by this generator are thrown over to another generator, which being lightly loaded, usually runs at a higher speed, that is higher frequency, and in consequence thereof all the synchronous motors operated on such a circuit are thrown out of step, which obviously is very objectionable, and the main objection against introduction of the single phase synchronous motor. The single phase induction motor, however, in such a case will keep running, since it does not depend upon exact synchronism but operates at a speed somewhat below synchronism.

MR. RIES:—What I had in view in putting the question was an application for the single phase motor which has not yet reached a practical stage to any extent. I had in mind the employment of single phase motors for electric railway work, long distance transmission and so on, in which alternating currents are used, with special reference to a system of secondary distribution employing a single supply and return conductor such as now used in direct current railways, and it was in that connection that I wanted a little further information from Mr. Steinmetz as to what particular stage the single phase induction motor has reached as compared with the well known synchron-

ous form. For stationary motors, where there is no particular difficulty in running a third wire, a two-phase or three-phase system of power distribution may have some advantages, at least under existing conditions. In the case of locomotors, however, the use of three separate conductors and sets of collecting devices for maintaining contact with them is productive of mechanical complications which may as well be avoided and which can best be overcome by the adoption of an efficient single-phase transmission system.

MR. STEINMETZ:—I do not see what I can say regarding that inquiry. As far as I know no single phase synchronous motor has ever been tried on a railway car and no single phase induction motor either as far as I know, and since it is much easier to use a direct current motor, and wherever the distance is great to transmit the power over a threephase transmission line by rotary converters, there has thus far been very little call for alternating railway motors, and the only promising field which I could see for them would be very heavy railway work or very high speed roads, that is 100 miles or more per hour, and other kinds of special work in which a polyphase induction motor, or perhaps even a single phase induction motor may perhaps be satisfactorily employed.

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DIED.

VERLEY:—At his home in Kingston, Jamaica, March 3rd, 1898, Horace S. L. Verley, son of Louis Verley of that place. Mr. Verley was formerly laboratory assistant of Dr. Geyer at the Stevens Institute, and was elected an associate member of the Institute May 17th, 1892. He had been ill for over a year.

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DIED.

PERRY:—In Brooklyn, N. Y., March 27th, 1898, Nelson Williams Perry, electrical and mining engineer. Mr. Perry was a son of Judge Perry of the Ohio Supreme Court, and was born in Columbus, Ohio, May 22nd, 1863. He joined the INSTITUTE May 17th, 1892, and was transferred to full membership March 21st, 1893. His death was due to the accidental drinking of a poisonous solution while working in his laboratory in darkness.

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